

# The Dynamics of Discrimination: Theory and Evidence

J. Aislinn Bohren, Alex Imas, Michael Rosenberg  
AER, 2019

Presentation: Ehsan Sabouri  
August 5, 2019

# Introduction

- Due to endogeneity issues, many researchers have employed field experiments to study discrimination. Field experiments have been successful in causally identifying the incidence of discrimination, but most cannot identify the source of this discrimination (**Bertrand and Duflo 2016**).
- One notable exception is **List (2004)**, who documents that minorities receive inferior initial and final offers when bargaining in a market for sports cards.
- This paper findings shed light on the mechanism behind previously documented discrimination reversals. for example, **Petersen and Saporta (2004)** find discrimination against women at the initial hiring stage for promotable jobs, but conditional on being hired, they find that women are more likely to be promoted.

# Description of Forum

## Mathematics Stack Exchange

- field experiment on an online Q&A mathematics forum, Mathematics Stack Exchange. Mathematics Stack Exchange is part of a family of forums where, in 2017 alone, 3,517,799 questions were asked and 4,299,077 answers were provided. (over 10 million registered users)

**How to bring  $5x_1^2 - 26x_1x_2 + 5x_2^2 + 10x_1 - 26x_2 = 31$  to the form  $\langle x', Ax' \rangle = 1$**

How can I bring

$$5x_1^2 - 26x_1x_2 + 5x_2^2 + 10x_1 - 26x_2 = 31$$

to the form

$$\langle x', Ax' \rangle = 1$$

where  $x' = \alpha x + \beta$  where  $\alpha \in \mathbb{R}^+$  and  $\beta \in \mathbb{R}^n$  in order to diagonalize  $A$ .

I tried to rewrite it to a vector and a matrix. But when I multiply it out I don't get the original equation.

Does anybody can help me?

Thank you a lot!

(linear-algebra) (abstract-algebra) (matrices) (vector-spaces)

share cite improve this question

Name

Reputation

asked Jul 14 at 23:44

Samuel

204 1 8

Figure 1: Question Post

# Description of Forum(continued)

## Mathematics Stack Exchange

- An upvote earns five reputation points on questions and ten reputation points on answers, while a downvote deducts two reputation points for both questions and answers.
- Reputation unlocks privileges, such as the ability to edit and comment on others posts or tag questions as duplicates. It can also be used as a currency through the assignment of "bounties".

Remember that complex solutions come in pairs when the coefficients of the polynomial are real, so  $z - 1 + i$  is also a factor. Since

6 ↑ Net Upvotes

$$(z - 1 - i)(z - 1 + i) = z^2 - 2z + 2,$$

you can divide  $z^4 + 3z^2 - 6z + 10$  by  $z^2 - 2z + 2$  to get a second degree polynomial. Then you can use the usual formula to solve the remaining second degree equation.

share cite improve this answer

edited Jan 9 '17 at 9:14

answered Jan 6 '17 at 16:19

Barbara

400 ■ 1 ▲ 14

Answer Accepted (+15)

Figure 2: Answer Post

# A Dynamic Model of Discrimination

# Model Setting

## Worker

Workers attribute :

- Each worker who has observable group identity  $g \in \{F, M\}$
- unobservable ability  $a \sim N(\mu_g, 1/\tau_a)$ , with mean  $\mu_g \in \mathbb{R}$  and precision  $\tau_a > 0$ .

The worker completes a sequence of tasks  $t = 1, 2, \dots$

Each task has hidden quality  $q_t = a + \varepsilon_t$  where  $\varepsilon_t \sim N(0, 1/\tau_\varepsilon)$  is an independent random shock with precision  $\tau_\varepsilon > 1$ .

Ability is fixed across time, and higher ability generates higher expected quality.

# Model Setting

## Evaluators

- A set of evaluators assess the worker's performance. For simplicity, assume that there is one evaluator per task, who reports evaluation  $v_t \in \mathbb{R}$ .
- Before evaluating task  $t$ , the evaluator observes:
  - ▶ the worker's gender  $g$
  - ▶ evaluations on past tasks  $h_t = (v_1, \dots, v_{t-1})$ , where  $h_1 = \emptyset$
  - ▶ signal  $s_t = q_t + \eta_t$  of quality of the current task, where  $\eta_t \sim N(0, 1/\tau_\eta)$  is an independent random shock with precision  $\tau_\eta > 0$

# Model Setting

## Evaluators

- A set of evaluators assess the worker's performance. For simplicity, assume that there is one evaluator per task, who reports evaluation  $v_t \in \mathbb{R}$ .
- Before evaluating task  $t$ , the evaluator observes:
  - ▶ the worker's gender  $g$
  - ▶ evaluations on past tasks  $h_t = (v_1, \dots, v_{t-1})$ , where  $h_1 = \emptyset$
  - ▶ signal  $s_t = q_t + \eta_t$  of quality of the current task, where  $\eta_t \sim N(0, 1/\tau_\eta)$  is an independent random shock with precision  $\tau_\eta > 0$
- An evaluator's type  $\theta_i$  determines her preferences and model of inference, including:
  - ▶ her subjective belief about the relationship between gender and ability
  - ▶ her subjective belief about other evaluators preferences and beliefs



# Model Setting

## Evaluators

- A set of evaluators assess the worker's performance. For simplicity, assume that there is one evaluator per task, who reports evaluation  $v_t \in \mathbb{R}$ .
- Before evaluating task  $t$ , the evaluator observes:
  - ▶ the worker's gender  $g$
  - ▶ evaluations on past tasks  $h_t = (v_1, \dots, v_{t-1})$ , where  $h_1 = \emptyset$
  - ▶ signal  $s_t = q_t + \eta_t$  of quality of the current task, where  $\eta_t \sim N(0, 1/\tau_\eta)$  is an independent random shock with precision  $\tau_\eta > 0$
- An evaluator's type  $\theta_i$  determines her preferences and model of inference, including:
  - ▶ her subjective belief about the relationship between gender and ability
  - ▶ her subjective belief about other evaluators preferences and beliefs
- The evaluator receives payoff  $-(v - (q - c_g^i))^2$  from reporting evaluation  $v$  on a task of quality  $q$  from a worker of gender  $g$ , where  $c_g^i$  is a type-specific taste parameter. Normalize  $c_M^i = 0$ .

# Model Setting

## Evaluators(continued)

**Definition 1** (Preference-Based Partiality). An evaluator of type  $\theta_i$  has a preference based partiality towards men if  $c_F^i > 0$ .

**Definition 2** (Belief-Based Partiality). An evaluator of type  $\theta_i$  has belief-based partiality towards men if  $\hat{\mu}_M^i > \hat{\mu}_F^i$ . This partiality is unbiased if  $\hat{\mu}_M^i = \mu_M$  and  $\hat{\mu}_F^i = \mu_F$  and otherwise is biased.

# Discrimination

Let

$$D_i(h, s) \equiv v_i(h, s, M) - v_i(h, s, F) \quad (1)$$

denote the difference between type  $\theta_i$ 's evaluation of a male and female worker conditional on observing history  $h$  and signal  $s$ , and let  $D(h, s) \equiv E_\pi [D_i(h, s)]$  denote the expected difference in evaluations across all types.

**Definition 3** (Discrimination). A woman faces discrimination from type  $\theta_i$  at  $(h, s)$  if  $D_i(h, s) > 0$ , and faces aggregate discrimination if  $D(h, s) > 0$ . A man faces (aggregate) discrimination if  $D_i(h, s) < 0$  ( $D(h, s) < 0$ ).

**Definition 4** (Discrimination Reversal). A discrimination reversal occurs at history  $h$  and signal  $s$  if there exists a history  $h' \subset h$  such that women face discrimination at  $(h', s)$  and men face discrimination at  $(h, s)$ .

# Initial Discrimination

- Consider the evaluation of the first task from a worker of gender  $g$  by an evaluator who has subjective prior beliefs  $(\hat{\mu}_F, \hat{\mu}_M)$  about average ability, preference parameter  $c_F$ , and observes signal  $s_1$ .
- The evaluator's prior belief about quality is normally distributed with mean  $\hat{\mu}_g$  and precision  $\tau_q \equiv \tau_a \tau_\varepsilon / (\tau_a + \tau_\varepsilon)$ , i.e.  $q_1 \sim N(\hat{\mu}_g, 1/\tau_q)$ .
- Given the prior belief and signal distribution, the evaluator's posterior belief about quality conditional on observing  $s_1$  is also normally distributed,  $q_1 | s_1 \sim N\left(\frac{\tau_q \hat{\mu}_g + \tau_\eta s_1}{\tau_q + \tau_\eta}, \frac{1}{\tau_q + \tau_\eta}\right)$ . the optimal evaluation is equal to:

$$v(h_1, s_1, g) = \frac{\tau_q \hat{\mu}_g + \tau_\eta s_1}{\tau_q + \tau_\eta} - c_g \quad (2)$$

Initial discrimination is independent of the signal and equal to

$$D(h_1, s_1) = \left( \frac{\tau_q}{\tau_q + \tau_\eta} \right) (\hat{\mu}_M - \hat{\mu}_F) + c_F \quad (3)$$

**Proposition 1** (Subjectivity of Judgement). If the evaluator has belief-based partiality, initial discrimination is decreasing in the precision of the signal  $\tau_\eta$  and otherwise, initial discrimination is constant with respect to  $\tau_\eta$ . As the signal becomes perfectly objective,  $\tau_\eta \rightarrow \infty$ , there is initial discrimination if and only if the evaluator has preference-based partiality.

# Dynamics of Discrimination

Two cases:

- ① evaluators share a common belief about the distribution of ability by gender, and this is common knowledge.
- ② evaluators have heterogeneous beliefs some evaluators have belief-based partiality towards men and believe that all evaluators share the same beliefs, while other evaluators have no belief-based partiality but are aware that some evaluators do.

# Dynamics of Discrimination

Suppose that all evaluators share a common prior belief about the distribution of ability by gender, have belief-based partiality, and this is common knowledge.

**Proposition 2** (Impossibility of Reversal). Suppose there is a single type of evaluator with belief-based partiality and no preference-based partiality. Then fixing an evaluation history, discrimination decreases across periods but never reverses.

# Dynamics of Discrimination

## Possibility of Reversal in Misspecified Model

**Proposition 3** (Possibility of Reversal). Suppose evaluators are the heuristic type  $\theta_1$  with probability  $p \in (0, 1)$  and the impartial type  $\theta_2$  with probability  $1 - p$ .

- 1 For any initial evaluation  $v_1$ , there exist cut-offs  $\bar{p} \in (0, 1)$  and  $\bar{s} \in \mathbb{R}$  such that for a low enough share of heuristic types  $p \in (0, \bar{p})$  and a high enough second period signal,  $s_2 > \bar{s}$ , aggregate discrimination reverses in the second period,  $D(v_1, s_2) < 0$ .
- 2 For any second period signal  $s_2$ , there exist cut-offs  $\bar{p}' \in (0, 1)$  and  $\bar{v} \in \mathbb{R}$  such that for a low enough share of heuristic types  $p \in (0, \bar{p}')$  and a low enough initial evaluation,  $v_1 < \bar{v}$ , aggregate discrimination reverses in the second period,  $D(v_1, s_2) < 0$ .



# Dynamics of Discrimination

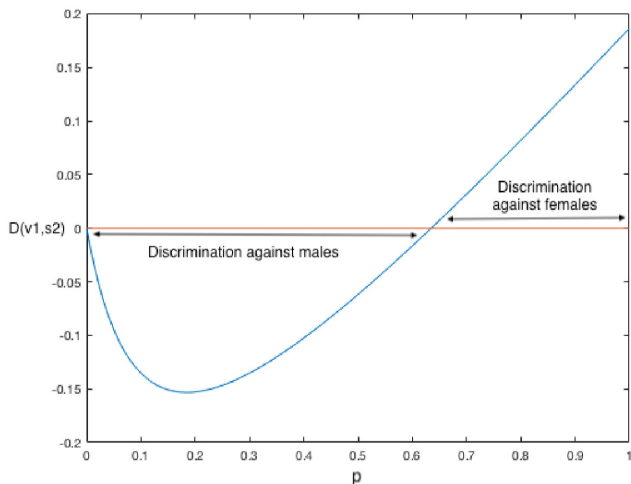


Figure 3: Second period discrimination, as a function of the proportion of heuristic evaluators.

# Field Experiment

---

# Experimental Design

## Posting Questions

- 280 new accounts, with 140 male usernames and 140 female usernames. 140 were left as new accounts; these comprised the Novice accounts.
- For the other 140 accounts, we manually built-up the reputation of at least 100.(Advanced accounts)
- we wrote 280 novel mathematics questions ranging in level of difficulty from upper-level undergraduate to early graduate.
- We measure discrimination as either the average change in reputation points per post (Rep) or the average number of upvotes net of downvotes per post (Net Votes).

# Experimental Design

## Posting Answers

- Original answers was generated to mathematics questions posted by other users on the forum, To examine how the subjectivity of judgment aects discrimination.
- Second set of 140 Novice accounts with no prior posts, split between 70 male usernames and 70 female usernames.
- research assistants worked in pairs('answerer' and 'poster')
- The standard of quality for answers is clear: determine whether or not the answer is correct. In contrast, there are multiple standards for judging the quality of a question.
- This difference in standards of quality should make judgment of questions more subjective than judgment of answers.

# Experimental Results

## Subjectivity of Judgment

Table 1: Effect of Gender on Evaluation of Novice Answers and Questions

	Answers Only		Questions Only		Answers & Questions	
	$\Delta$ Rep	Net Votes	$\Delta$ Rep	Net Votes	$\Delta$ Rep	Net Votes
	(1)	(2)	(3)	(4)	(5)	(6)
Male	-1.38 (.97)	-0.31 (.17)	2.86 (1.32)	0.58 (.27)	-1.38 (1.16)	-0.31 (.22)
Question					0.08 (1.16)	0.09 (.22)
Male*Question					4.24 (1.64)	0.89 (.32)
Constant	4.60 (.69)	0.79 (.12)	4.68 (.93)	0.88 (.19)	4.60 (.96)	0.79 (0.16)
# Obs	135	135	135	135	270	270

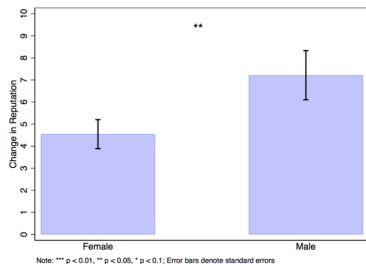
# Experimental Results

## Subjectivity of Judgment

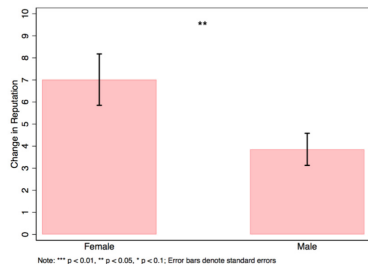
- We find significant *initial discrimination* against females: Questions posted to accounts with female usernames accumulated significantly fewer reputation points .
- There is a significant mitigation of discrimination against female accounts for answers, relative to questions: the interaction effect between gender and type of post is positive and significant in both specifications.

Taken together, these results are inconsistent with discrimination due to preference based partiality. Rather, they support the theoretical prediction on how subjectivity affects discrimination when evaluators have *belief-based* partiality.

# Experimental Results



(a) Novice Accounts



(b) Advanced Accounts

Figure 4: Average Reputation Difference for Questions

# Experimental Results

## Dynamics of Discrimination

Table 2: Effect of Gender on Evaluation Questions, Novice and Advanced

	Advanced		Novice & Advanced		
	$\Delta$ Rep	Net Votes	$\Delta$ Rep	Net Votes	Binary
	(1)	(2)	(3)	(4)	(5)
Male	-3.16 (1.37)	-0.62 (.28)	2.86 (1.36)	0.58 (.27)	0.17 (.08)
Advanced			2.33 (1.35)	0.49 (.27)	0.09 (0.08)
Male*Advanced			-6.02 (1.91)	-1.20 (.38)	-0.40 (.11)
Constant	7.01 (0.97)	1.38 (.20)	4.68 (.96)	0.88 (0.19)	0.56 (.56)
# Obs	138	138	273	273	273



# Experimental Results

## Dynamics of Discrimination

- Questions posted to advanced female accounts accumulated more reputation points,  $\Delta\text{Rep}$ , than those posted to advanced male accounts:  
There is a dynamic *reversal of discrimination* between novice and advanced accounts.
- In our setting, not only is initial discrimination against females mitigated by reputation, but the direction of discrimination reverses, females are favored at higher reputations

Interpreting these findings through the lens of the theoretical framework, our results suggest that initial discrimination is driven by belief based partiality with bias.

# Discussion and Conclusion

Three main results are:

- ① significant gender discrimination exists at the initial stage, in the form of less reputation earned per post and fewer votes per post on questions posted by low reputation female accounts relative to questions posted by low reputation male accounts.
- ② significantly less gender discrimination at the initial stage for answers, where judgment of quality is less subjective relative to questions.
- ③ discrimination reverses for questions at more advanced stages, in that more reputation is earned and more votes are received on questions posted to high reputation female accounts relative to high reputation male accounts.

Thank You