Gravity Trade Models: ***an Overview***

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A Bit of History

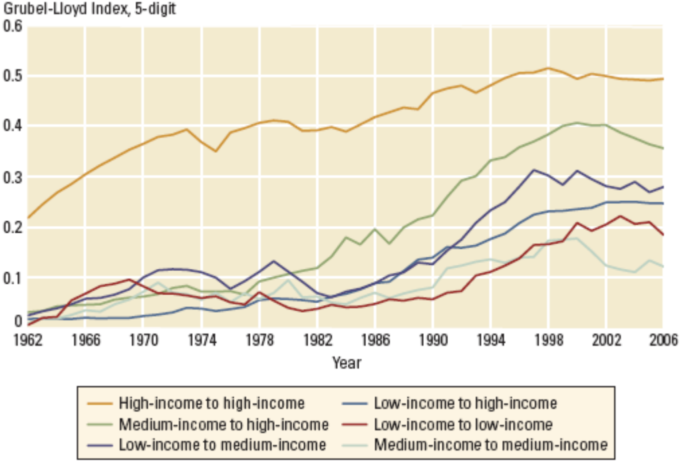
Early 1900s

- Once upon a time *Comparative advantage* looked pretty good as a description of trade.

Late 1900s

- … but trade patterns transformed over time: countries exported the same goods they imported!

- The rise of intraindustry trade.



Source: Brülhart 2008 for this Report.
Note: The Grubel-Lloyd index is the fraction of total trade that is accounted for by intraindustry trade.

What Drives Within-Industry Trade?

A straightforward explanation: ***Product Differentiation.***

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Many other explanations based on:

1. Increasing returns to scale (Krugman 1980)
2. Comparative cost advantage (Eaton-Kortum 2002)
3. Firm heterogeneity (Chaney 2008)

A New Generation of Trade Models

- Many countries: 1,...,N
- Many industries: 1,..., K
- Trade *across industries* driven by comparative advantage.
- Trade *within industries* driven by forces of ***gravity.***

Many micro-foundations, one equation!

The Gravity Equation

The *gravity equation* describes bilateral trade values within industry k:

$$X_{ji,k} = \frac{\left(\tau_{ji,k} w_j / A_{j,k}\right)^{-\theta_k}}{\sum_n \left(\tau_{ni,k} w_n / A_{n,k}\right)^{-\theta_k}} E_{i,k}$$

- $X_{ji,k}$: Exports sales from ***country*** j ***to*** i in ***industry*** k

The Gravity Equation: *Elements*

The *gravity* equation:

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- w_j: wage rate

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- $E_{i,k}$: country i's total spending on sector k
- C-D utility across sectors $\implies E_{i,k} = \alpha_{i,k} Y_i$
- Total income: $Y_i =$ wage $\times$ population size $= w_i L_i$

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- $A_{j,k}$: country j's efficiency in sector k
- Two components: $A_{j,k} = T_{j,k} L_{j,k}^{\psi_k}$
    1. $L_{j,k}^{\psi_k}$: scale effects ($L_{j,k}$: size of sector-level labor force)
    2. $T_{j,k}$: other factors (e.g., human capital)

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Two key parameters:

- θ_k: trade elasticity
- ψ_k: scale elasticity

First, let's put the gravity model in perspective.

The Gravity Model in Perspective

- The world economy:
  - 196 countries
  - 16 tradable industries (WIOD classification)
- The gravity equation characterizes a 196×196 matrix of trade values for each of the 16 sectors.

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The Gravity Model in Perspective: *An Example*

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- ... in some industries trade is rather balanced:

Medical Eq: $X_{US \rightarrow EU, MED.} \approx X_{EU \rightarrow US, MED.} = \$26B$

The Gravity Model in Perspective: *An Example*

- Consider the US-EU trade.
- The gravity equation will predict intra-industry trade in all 16 industries, but...
- ... in some industries the EU is a net exporter:

Machinery: $X_{EU \rightarrow US, MCH.} = \$70B > X_{US \rightarrow EU, MCH.} = \$31B$

The Gravity Model in Perspective: *An Example*

- Consider the US-EU trade.

- The gravity equation will predict intra-industry trade in all 16 industries, but...

- ... in some industries the EU is a net importer:

Aircrafts: $X_{US \rightarrow EU, AIR.} = \$35B > X_{EU \rightarrow US, AIR.} = \$2B$

One Model, 2 Types of Trade

1. Within industry trade (governed by θ_k)

$$\text{lower } \theta_k \implies \text{more within-industry trade}$$

2. Across industry trade (governed by $A_{j,k}$)

$$\frac{A_{1,a}}{A_{2,a}} > \frac{A_{1,b}}{A_{2,b}} \implies \text{country 1 net exporter of industry "a" to 2}$$

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A special case: $\theta_k \longrightarrow \infty$

- No within-industry trade.
- The gravity framework reduces to a standard neoclassical trade model

How can we compute and asses the predictions of the gravity models?

- **First:** define the equilibrium.
- **Second:** calibrate the model

Equilibrium

- **Exogenous components:**
  - Deep parameters: $\boldsymbol{\theta} \equiv \{\theta_k\}$, $\boldsymbol{\psi} \equiv \{\psi_k\}$
  - Policy variables: $\boldsymbol{\tau} \equiv \{\tau_{ji,k}\}$, $\boldsymbol{L} \equiv \{L_j\}$, $\boldsymbol{T} \equiv \{T_{j,k}\}$
- **Eq. outcome:** $\boldsymbol{w} \equiv \{w_j\}$
- **Eq. condition:** $w_i L_i = \sum_k \sum_i X_{ji,k}(\boldsymbol{w}; \boldsymbol{\tau}, \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{L}, \boldsymbol{T})$

Calibration Strategy

- $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ require micro-level estimation.
- $\boldsymbol{L}$, $\boldsymbol{w}$, and $\boldsymbol{X}$ are observable.

Calibration Goal:

- Choose $\boldsymbol{\tau}$ and $\boldsymbol{T}$ ($N \times N \times K + N$ parameters)
- Match $\boldsymbol{X}$ and $\boldsymbol{w}$ ($N \times N \times K + N$ data points)

Calibration Strategy

- On paper, gravity models can exhibit a *perfect* fit...
- ... but "in practice" we prefer $\boldsymbol{\tau}$ to have some structure.

Typically, researchers assume:

$$\tau_{ji,k} = \beta_k \left(Dist_{ji}\right)^{\beta_{D,k}} \left(Border_{ji}\right)^{\beta_{B,k}} \left(Lang_{ji}\right)^{\beta_{L.k}}$$

- $\boldsymbol{\tau}$ is characterized by $4 \times K$ parameters $\boldsymbol{\beta}$

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Calibration Strategy

The calibration problem can be stated as

$$\min_{\boldsymbol{\beta},\boldsymbol{T}} \sum_k \sum_{j,i} \left(\hat{X}_{ji,k}(\boldsymbol{T}, \boldsymbol{\beta}; \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{L}, \boldsymbol{w}) - X_{ji,k} \right)^2$$

$$s.t. \;\; w_i L_i = \sum \hat{X}_{ji,k}(\boldsymbol{T}, \boldsymbol{\beta}; \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{L}, \boldsymbol{w})$$

Multiple ways of handling the problem:

- The structural approach (Anderson-Van Wincoop 2003, Fieler 2011)
- The MPEC approach (Balistreri et al. 2011)
- The reduced form PPML approach (Santos Silva-Tenreyro 2006)

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Calibration Strategy: *Structural Approach*

Inner loop:

- Fix $\boldsymbol{\beta}$
- For each $\boldsymbol{T}$ we can compute $\hat{\boldsymbol{X}}(\boldsymbol{T}, \boldsymbol{\beta}; \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{L}, \boldsymbol{w})$
- Solve for $\boldsymbol{T}$ that satisfies

$$w_i L_i = \sum \hat{X}_{ji,k}(\boldsymbol{T}, \boldsymbol{\beta}; \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{L}, \boldsymbol{w})$$

Outer loop:

- Search for $\boldsymbol{\beta}$ that minimize $| \hat{\boldsymbol{X}}_{\text{inner loop}} - \boldsymbol{X} |$.

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Calibration Strategy: *Reduced Form*

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Calibration Strategy: *Reduced Form*

The gravity equation

$$\ln X_{ji,k} = \theta_k \ln \beta_k + \theta_k \beta_{D,k} \ln Dist_{ji} + EX_{j,k} + IM_{i,k} + \varepsilon_{ji,k}$$

Challenge:

Plain OLS $\Longrightarrow EX_{j,k}$ and $IM_{i,k}$ may be inconsistent with *"Balanced Trade"*.

Calibration Strategy: *Reduced Form*

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Solution?

Use a PPML estimator (Fally 2013).

Goodness of Fit

Structural approach:

- Good fit when sample includes only rich countries.
- Poor fit when sample includes rich & poor countries.

Reduced form approach:

- Importer FE offers an additional degree of freedom $\Longrightarrow$ better fit (similar to a *non-homothetic* structural model).

Out-of-sample performance: Not great!

Implications

- Gravity models are used to answer *policy* questions.

- One question has attracted the most attention.

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Computing the Gains from Trade

One approach:

- Counterfactually set $\boldsymbol{\tau} \rightarrow \infty$ in the calibrated model
- Calculate the change in real income per worker.

However...

- The gains from trade can be calculated without performing the full calibration (Arkolkais-Costinot-Rodriguez Clare 2011).

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To demonstrate the ACR approach,
let's start with a basic *one-sector* economy.

The Gains from Trade

Real income per worker can be state as:

$$W_i = T_j \times L_j^{\psi} \times \lambda_{ii}^{-\frac{1}{\theta}} \times \tau_{ii}^{-1}$$

The Gains from Trade

Population Size

Trade

$$W_i = T_j \times L_j^{\psi} \times \lambda_{ii}^{-\frac{1}{\theta}} \times \tau_{ii}^{-1}$$

Efficiency

dom. trade frictions

The Gains from Trade

The welfare effects of reducing international trade costs.

$$\hat{W}_i = \hat{\lambda}_{ii}^{-\frac{1}{\theta}}$$

- Hat notation: $\hat{x} \equiv \frac{x'}{x}$

The Gains from Trade

$$GT_i \equiv \frac{W_i}{W_i^A} = \left(\frac{\lambda_{ii}}{\lambda_{ii}^{\text{A}}}\right)^{-\frac{1}{\theta}}$$

- in autarky $\lambda_{ii}^A = 1$.
- θ can be estimated with micro-level data.

The Gains from Trade

$$GT_i = \lambda_{ii}^{-\frac{1}{\theta}}$$

- λ_{ii} is directly observable.
- θ can be estimated with micro-level data.

The Gains from Trade (Year 2008, $\theta = 5$)

	λ_{ii}	% GT
Ireland	0.68	8%
Belgium	0.70	7.5%
Germany	0.80	4.5%
China	0.88	2.6%
U.S.	0.92	1.8%

- ***Iran:*** $\lambda_{ii} = 0.8$, $GT = 4.6\%$

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First Extension: Allowing for Intermediate Trade

Allowing for Intermediate Inputs

Simplest way:

- Production combines labor and intermediates.
- $\boldsymbol{\beta} \in (0,1)$: share of labor in production
- Price of Intermediates inputs = consumer price index $\equiv \boldsymbol{P_i}$

$$X_{ji} = \frac{\left(\tau_{ji,k} w_j^{\beta} P_j^{1-\beta} / A_{j,k}\right)^{-\theta_k}}{\sum_n \left(\tau_{ni,k} w_n^{\beta} P_n^{1-\beta} / A_{n,k}\right)^{-\theta_k}} E_{i,k}$$

$$GT_i = \lambda_{ii}^{-\frac{1}{\beta\theta}}$$

The Gains from Trade (Year 2008, $\theta = 5$, $\beta = 0.5$)

		% GT	
	λ_{ii}	baseline	intermediates
Ireland	0.68	8%	16.6%
Belgium	0.70	7.5%	15.6%
Germany	0.80	4.5%	9.2%
China	0.88	2.6%	5.3%
U.S.	0.92	1.8%	3.6%

Second Extension: Multiple Sectors

Gains from Trade: *Multiple Sectors*

Real income per worker

$$W_i = T_i \, \pi_{ii}^{-1} \left(\prod_s L_{i,s}^{\beta_{i,s} \boldsymbol{\psi_s}} \right) \left(\prod_s \lambda_{ii,s}^{-\frac{\beta_{i,s}}{\boldsymbol{\theta_s}}} \right)$$

Gains from Trade: *Multiple Sectors*

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Gains from Trade: *Multiple Sectors*

$$GT_i = \underbrace{\prod_k \hat{L}_{i,k}^{\alpha_{i,k}\boldsymbol{\psi_k}}}_{scale-driven} \prod_k \lambda_{ii,k}^{-\frac{\alpha_{i,k}}{\boldsymbol{\theta_k}}}$$

First, consider a competitive model:

- $\psi_k = 0 \implies$ *scale-driven term*$=0$
- We only need the sector-level trade elasticities: θ_k

Multiple Sectors + No Scale Effects

expenditure share

$$GT_i = \prod_k \lambda_{ii,k}^{-\frac{\alpha_{i,k}}{\theta_k}}$$

trade intensity

trade elasticity

Multiple Sectors + No Scale Effects

	% GT	
	one-sector	multi-sector
Ireland	8%	23.5%
Belgium	7.8%	32.7%
Germany	4.5%	12.7%
China	2.6%	4%
U.S.	1.8%	4.4%

Now, consider a model with scale effects:

- Gains also depend on sector-level *scale elasticities*, $\psi_{\mathbf{k}}$.

- High-ψ industries $\implies$ stronger scale economies $\implies$ greater returns to specialization.

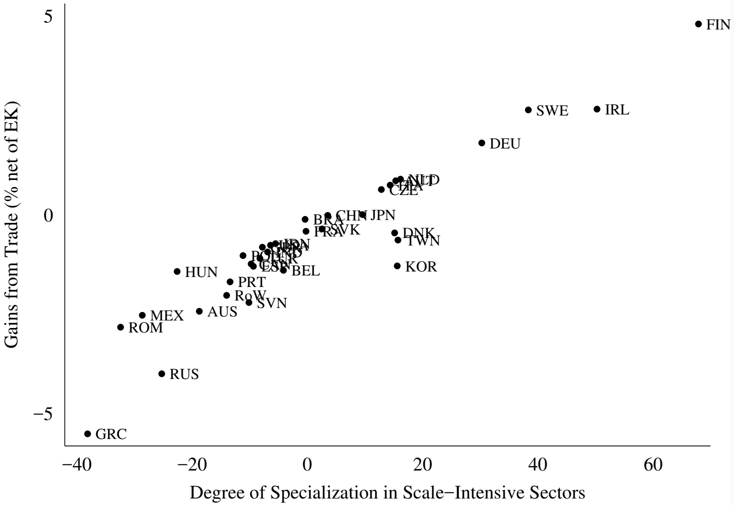
- Trade favors countries that specialize in high-ψ industries

Multiple Sectors + Scale Effects

$$GT_i = \prod_k \hat{L}_{i,k}^{\alpha_{i,k}\boldsymbol{\psi_k}} \prod_k \lambda_{ii,k}^{-\frac{\alpha_{i,k}}{\boldsymbol{\theta_k}}}$$

- $\hat{L}_{i,k} \equiv \frac{\text{factual employment}}{\text{autarky employment}} = \frac{r_{i,k}}{\alpha_{i,k}}$
- $r_{i,k}$: share of revenue generated in sector k

Gains from Trade × Sectoral Specialization



Last Extension: Multiple Factors

Finally, consider a model with multiple factors of production.

- Labor market structure:
  - Different groups of workers: *indexed by g*.
  - Roy model of industry choice.
  - Group-wide abilities vary across sectors.
  - Within-group heterogeneity.

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Gains from Trade: *Multiple Factors*

$$\hat{W}_{ig} = \prod_k \hat{\lambda}_{ii,k}^{-\frac{\alpha_{i,k}}{\theta_k}} \underbrace{\prod_k \hat{\pi}_{ig,k}^{-\frac{\alpha_{i,k}}{\eta}}}_{\textbf{distributional effects}}$$

- η: elasticity of labor supply.

Multiple Factors: *Special Case1*

$\eta \to \infty$: ***standard one factor model***

$$\hat{W}_{ig} = \prod_k \hat{\lambda}_{ii,k}^{-\frac{\alpha_{i,k}}{\theta_k}}$$

Multiple Factors: *Special Case1*

$\eta \to \infty$: ***standard one factor model***

$$\frac{\hat{W}_{ig}}{\hat{W}_i} = 1$$

- No distributional effects.

Multiple Factors: *Special Case 2*

$\eta \to 1$: ***specific factor model***

$$\frac{\hat{W}_{ig}}{\hat{W}_i} = \prod_k \pi_{ig,s} \hat{r}_{i,s}$$

change in industry size

Main insight:

- Trade favors groups employed intensively in export sectors

Multiple Factors: *Special Case 2*

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- ...so is the vast majority of the literature.
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