Adverse Selection and Market Design

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Introduction

- Much of last lecture focused on private value problems: valuation of the seller and the buyer are uncorrelated

- Many markets are the opposite: valuations of buyers and sellers are perfectly correlated:
  - Used Cars: classic example
  - Insurance: value of insurance to the buyer depends on risk - accident, heart attack, death - which determines the cost to the seller; information is private to the buyer
  - Asset market: some traders know more about the value of a security; underwriter of a mortgage backed security knows more about the underlying mortgages

- Key issue: how to organize decentralized trade and allow for competition

Outline

- Discuss basic models of adverse selection and decentralized trade
- Some applications to insurance and finance
- Some empirics

A Basic Model of Trade

- Goods:
  - Two goods: one indivisible good; one divisible numeraire
- Players:
  - Two principals with valuation of the indivisible good: $v(c)$
  - One agent with valuation of the indivisible good: c
- Common value assumption: $v'(c) > 0$
- Asymmetric information: $c \sim F(c)$; privately known by the agent
- $v(c) > c$: Full trade optimal under full information

A Basic Model of Trade

- Timing:
  - Principals make price offers
  - Agent makes a choice

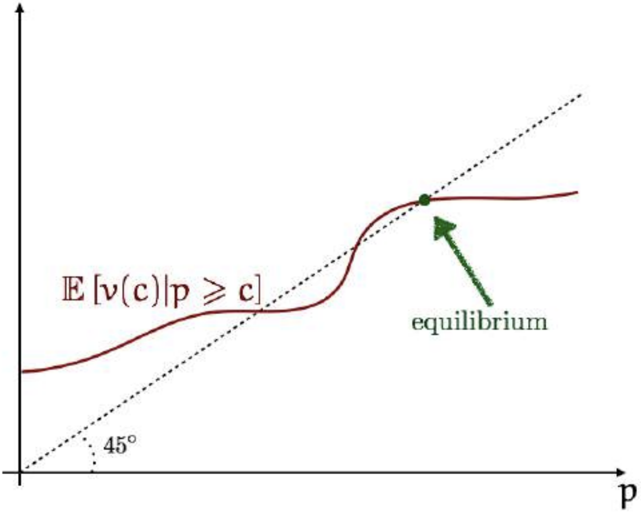
- Agent's choice: choose the higher price

$$\text{sell} \Leftrightarrow \max\{p_1, p_2\} \geqslant c$$

- In equilibrium:
  - prices are equal: $p_1 = p_2$.
  - Profits are zero:

$$p = \mathbb{E}\left[v(c)|c \leqslant p\right]$$

Equilibrium



Equilibrium

- The precise nature of the equilibrium depends on $F(c)$ and gains from trade $v(c) - c$.
- If gains from trade are large relative to values of c then everyone trades;
- If gains from trade for low c are small, it is possible that all trades break down - Akerlof's original example
- Note: Equilibrium is (constrained) efficient; a central planner that is subject to the same information constraint cannot improve upon this equilibrium

Product Design

- Adverse selection: good types are excluded (even though it is constrained efficient)
  - Insurance: low-risk individuals are excluded

- The 0-1 nature of trade leads to exclusion.

- Perhaps allowing for new products can lead to more trade:
  - perhaps low-risk individuals are willing to accept less coverage in return for a lower price/premium

Product Differentiation

- Principals offer products of the form $\{x, p(x)\}_{x \in [0,1]}$

- Payoffs:

$$\text{Agent: } xp(x) + c(1-x)$$

$$\text{Principal: } v(c)x - xp(x)$$

- Interpretation of x:
  - insurance: level of coverage; copay
  - finance: retention of mortgage backed securities; quantity traded

- choice of agent: $x^*(c)$

Equilibrium

- Result 1: Any equilibrium must be separating, i.e., $x^*(c)$ is one-to-one
  - Idea of proof: If not then a principal is making money on some types and a competitor can come in and target those types only

- Result 2: $p(x^*(c)) = v(c)$, i.e., profits are zero type by type

- Result 3: $x^*(c)$ must satisfy

$$x(\underline{c}) = 1; x^*(c) \frac{v'(c)}{(x^*)'(c)} + v(c) = c$$

- Note: $(x^*)'(c) < 0$.

Equilibrium does not exist!!!!!

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- There always exists a mixed-strategy equilibrium: always inefficient!

Market Design Question

- Can we come up with market mechanisms that allow for cross-subsidization?

- Caveats in the equilibrium concept:
  - agent can costlessly switch between principals
  - principals can costlessly create products
  - one-agent assumption: no interdependence between terms offered to different types of agents.

- I will talk about relaxing some of these assumptions: starting with switching costs for the agent

Agent's Switching Costs

- Based on joint paper with Lester, Venkateswaran and Zetlin-Jones (2016)

- Suppose with some probability the agent can only trade with one of the principals

$$\mathbb{P}(\text{can trade with both}) = \pi; \mathbb{P}(\text{can trade only with i}) = \frac{1-\pi}{2}$$

- For simplicity $c \in \{c_l < c_h\}$

- Principals do not know the agent's trading opportunities: Have to offer the same terms to captive and non-captive individuals

Structure of Equilibrium

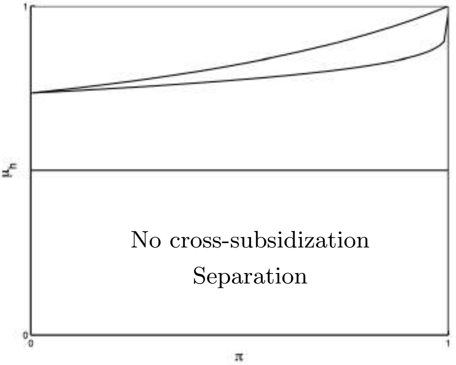
- Result 0: There is no pure strategy equilibrium
  - principals can always guarantee positive profits by targeting a captive agent; At the same time like to increase prices to attract a non-captive agent
  - Principals mix over menus: (x_l, p_l, x_h, p_h)
    - Can summarize each menu with the vector of utilities to each type (u_l, u_h)
  - Equilibrium distribution of utilities $F_l(u_l), F_h(u_h)$

Switching Costs: Results

- Result 1: terms of trades are positively correlated across types: more generous contracts to high types are also generous for low types

- Result 2: Depending on structure of competition and adverse selection contracts can be cross-subsidizing or not.

Equilibrium Contracts



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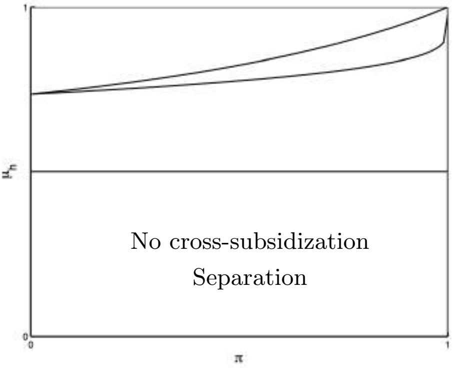

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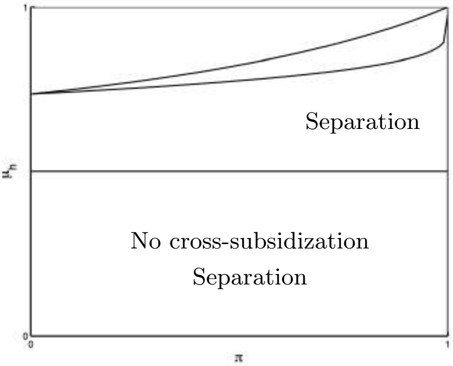
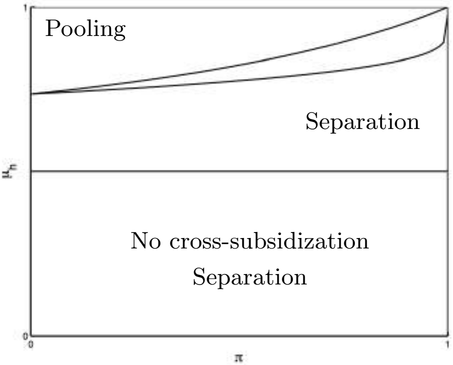


Cross-subsidization equilibrium may feature:

- Full Separation: $0 < x_h < 1$ a.e.

Equilibrium Contracts



Cross-subsidization equilibrium may feature:

- Full Separation: $0 < x_h < 1$ a.e.
- Full Pooling: $x_h = 1$ a.e.

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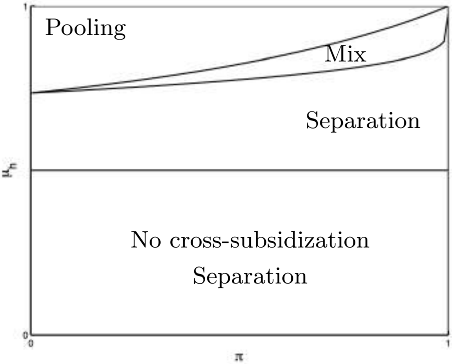

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- Mix: pool below $\bar{u}_l$, separate above

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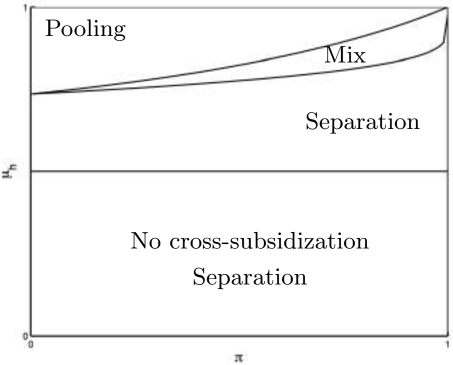

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More competition (higher π) $\rightarrow$ *less* pooling

- gains to separation increase in π

Milder adv sel (higher μ_h) $\rightarrow$ *more* pooling

- increased incentives to trade with h

Switching Costs: Results

- Result 1: terms of trades are positively correlated across types: more generous contracts to high types are also generous for low types
- Result 2: Depending on structure of competition and adverse selection contracts can be cross-subsidizing or not.
- Result 3: When $\mathbb{P}(c_h)$ is low, welfare is maximized at $\pi \in (0, 1)$
    - With π close to 1, competition is intense for high types which leads to high prices for them and low quantities (in order to keep low types from choosing a high price contract!)

Application to Financial Markets

Application to Financial Markets

- Some details first
- What is our plain vanilla model of securities trading?
  Lucas-Breeden - Stock markets are Walrasian:
  - There is a single price!
  - agents take it as given.
  - they can trade without limits
- How does it work in practice?

Financial Markets

- There is no single price: Typical buy and sell prices (ask and bid) are different; large trades are also typically executed away from small trades (dark pools)
- Large trades do not necessarily have the same price as small trades: traders order size affect prices
- Short-selling is typically subject to borrowing limits

Financial Markets Design

- A lot of securities - stocks, options, ETFs - trade on electronic limit order book.

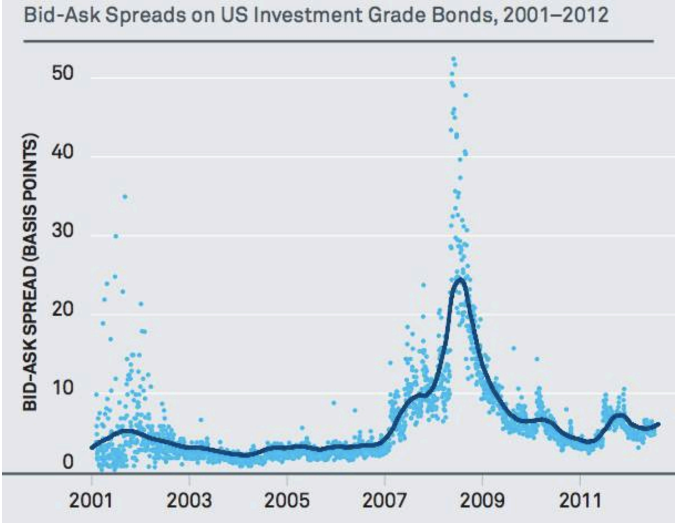
- People submit:
  - limit order: sell (buy) quantity q at any price above (below) p until time t
  - market order: sell (buy) at the best available price

Limit Order Book

Market Liquidity

- Liquidity: ease at which you can execute an order
- Bid-Ask spread is a measure of market liquidity

Market Liquidity



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    - OTC Market models: Duffie, Garleanu and Pedersen (2005)
  - buyers and sellers have private information about the true value of the security
    - Glosten and Milgrom (1984)

Glosten and Milgrom, 1984

- A simplified version here
- A security that pays $V \in \{V_L < V_H\}$ at the end of the day with $\mathbb{E}V = \overline{V}$ and $\Pr(V = V_H) = \mu$.
- Three sets of agents:
  - Two dealers or market makers: risk neutral and uninformed
  - Fraction λ of informed traders (I): they know V
  - Fraction $1 - \lambda$ of liquidity traders (L): uninformed, and would like to sell or buy (with equal probability) no matter the quoted price
- Everyone has a unit demand

Glosten and Milgrom, 1984

- Traders arrive uniformly and randomly
- Dealers quote two prices: bid, B - the price at which they buy and ask, A - the price at which they sell
- Competition among dealers means profits are zero, i.e.,

$$\begin{aligned} A &= \mathbb{E}[V|\text{submitted order} = \text{buy}] \\ B &= \mathbb{E}[V|\text{submitted order} = \text{sell}] \end{aligned}$$

Glosten and Milgrom, 1984

- Result 1: V_H: I buys, V_L: I sells

- Result 2: Bid and Ask:

$$
\begin{aligned}
\mathbb{P}(\text{buy}) &= \lambda\mu + (1-\lambda)\frac{1}{2} \\
A = \mathbb{E}(V|\text{buy}) &= \frac{\lambda\mu}{\lambda\mu + (1-\lambda)\frac{1}{2}} V_H + \frac{(1-\lambda)\frac{1}{2}}{\lambda\mu + (1-\lambda)\frac{1}{2}} \overline{V} \\
B = \mathbb{E}(V|\text{sell}) &= \frac{\lambda(1-\mu)}{\lambda(1-\mu) + (1-\lambda)\frac{1}{2}} V_L + \frac{(1-\lambda)\frac{1}{2}}{\lambda(1-\mu) + (1-\lambda)\frac{1}{2}} \overline{V}
\end{aligned}
$$

    ◦

Glosten and Milgrom, 1984

- Once the order arrives, the dealers update their beliefs

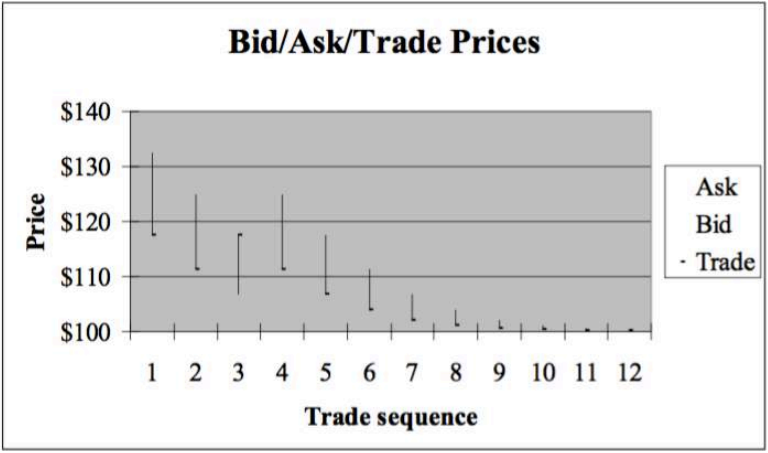
$$\mu'(\text{sell}) = \frac{\mu}{\lambda\mu + (1-\lambda)\frac{1}{2}}$$

$$\mu'(\text{buy}) = \frac{\mu(1-\lambda)}{\lambda(1-\mu) + (1-\lambda)\frac{1}{2}}$$

- Result 3:

$$\lim_{t\to\infty} A_t - B_t = 0$$

Glosten and Milgrom, 1984



Glosten and Milgrom, 1984

- It is possible to extend this along various dimensions:
  - Make liquidity traders price sensitive
  - More general stochastic process for the payoff of the stock
  - Difference in order sizes
- Empirical work: Easley and O'hara try to measure how much trade is due to asymmetric information

Finance Market Design Issues

- Liquidity: Search and bargaining vs private information; Lester, Shourideh, Venkateswaran, Zetlin-Jones (2016)
- High Frequency Trading: Trades are executed in the micro second level; some argue that HFT's try to take advantage of market orders by front-running them: Budish, Cramton, Shim, 2015.
- Number of trading platforms - exchanges - have increased a lot; leading to regulatory complications of how orders should be executed
- Transparency of prices in Over-the-Counter markets

Application to Insurance

Based on joint work with Chari and Zetlin-Jones

Players

- Continuum of households of unit mass:
  - ⋆ low risk (good) and high risk (bad): $j \in \{g, b\}$
  - ⋆ endowment: $\omega \in \{\omega_2 < \omega_1\}$; 2: loss state
    - risk: $\Pr(\omega_1 | j) = \pi_j ; \pi_g > \pi_b$
  - ⋆ Population fractions: $\Pr(j) = \mu_j : \mu_g + \mu_b = 1$
  - ⋆ Concave utility function $u(c)$
- 2 risk-neutral insurance companies (firms)

Allocations, Payoffs, ...

- Allocations: $\mathbf{c} = (c_{1j}, c_{2j})_{j \in \{g,b\}}$
- Payoffs:
  - ⋆ Households:

$$U_j(\mathbf{c}) = \pi_j u(c_{1j}) + (1 - \pi_j) u(c_{2j})$$

  - ⋆ Firms - from type j:

$$\pi_j(\omega_1 - c_{1j}) + (1 - \pi_j)(\omega_2 - c_{2j})$$

  - ⋆ Total firms profits $\Pi^i(\mathbf{c})$

Incentive Compatibility

- Risk types: private information to the household
- Focus on direct mechanisms: $(c_{1g}, c_{2g}, c_{1b}, c_{2b})$
- Incentive compatibility:

$$
\begin{aligned}
\pi_b u(c_{1b}) + (1-\pi_b)u(c_{2b}) &\geqslant \pi_b u(c_{1g}) + (1-\pi_b)u(c_{2g}) \\
\pi_g u(c_{1g}) + (1-\pi_g)u(c_{2g}) &\geqslant \pi_g u(c_{1b}) + (1-\pi_g)u(c_{2b})
\end{aligned}
$$

- relevant IC: *b* pretending to be *g*

Efficiency

- Notion of efficiency: low risk efficient
  - Max welfare of g subject to
    - IC
    - resource constraint
    - participation by b : must be better off than autarkic full insurance
  - Natural candidate for equilibrium

Autarkic Full Insurance

- Autarkic full insurance

$$
\begin{aligned}
V_b^f &= \max_{c_1, c_2} \pi_b u(c_1) + (1 - \pi_b) u(c_2) \\
& \text{subject to} \\
& \pi_b(\omega_1 - c_1) + (1 - \pi_b)(\omega_2 - c_2) \geqslant 0
\end{aligned}
$$

Low-Risk Efficiency

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- For any composition of types

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- For any composition of types (λ_b, λ_g)

$$V_g^{eff}(\lambda_b, \lambda_g) = \max_{(c_{1j}, c_{2j})} \pi_g u(c_{1g}) + (1 - \pi_g) u(c_{2g})$$

subject to

$$\pi_b u(c_{1b}) + (1 - \pi_b) u(c_{2b}) \geqslant \pi_b u(c_{1g}) + (1 - \pi_b) u(c_{2g})$$

$$\pi_b u(c_{1b}) + (1 - \pi_b) u(c_{2b}) \geqslant V_b^f$$

Low-Risk Efficiency

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- Equivalently defines $V_b^{eff}(\lambda_b, \lambda_g)$

Efficient Allocations

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Efficient Allocations

- utilities are homogenous of degree 0 in (λ_b, λ_g)
- If $\frac{\lambda_g}{\lambda_g + \lambda_b} \leqslant \lambda^*$ then
  - ⋆ least-cost-separating allocation
  - ⋆ participation constraint binds
  - ⋆ incentive constraint binds
  - ⋆ no cross-subsidization; profits are zero on each type

Efficient Allocations

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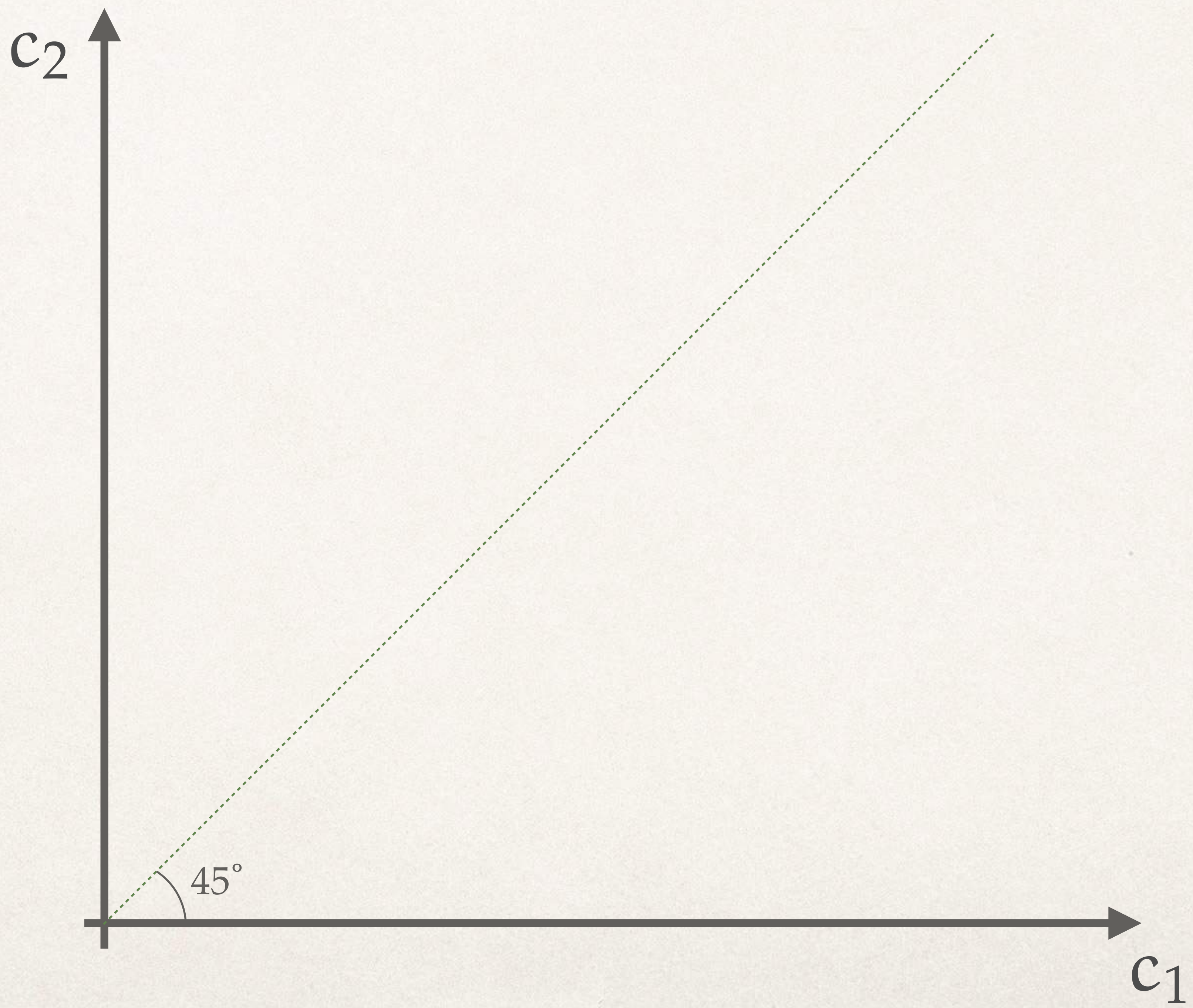

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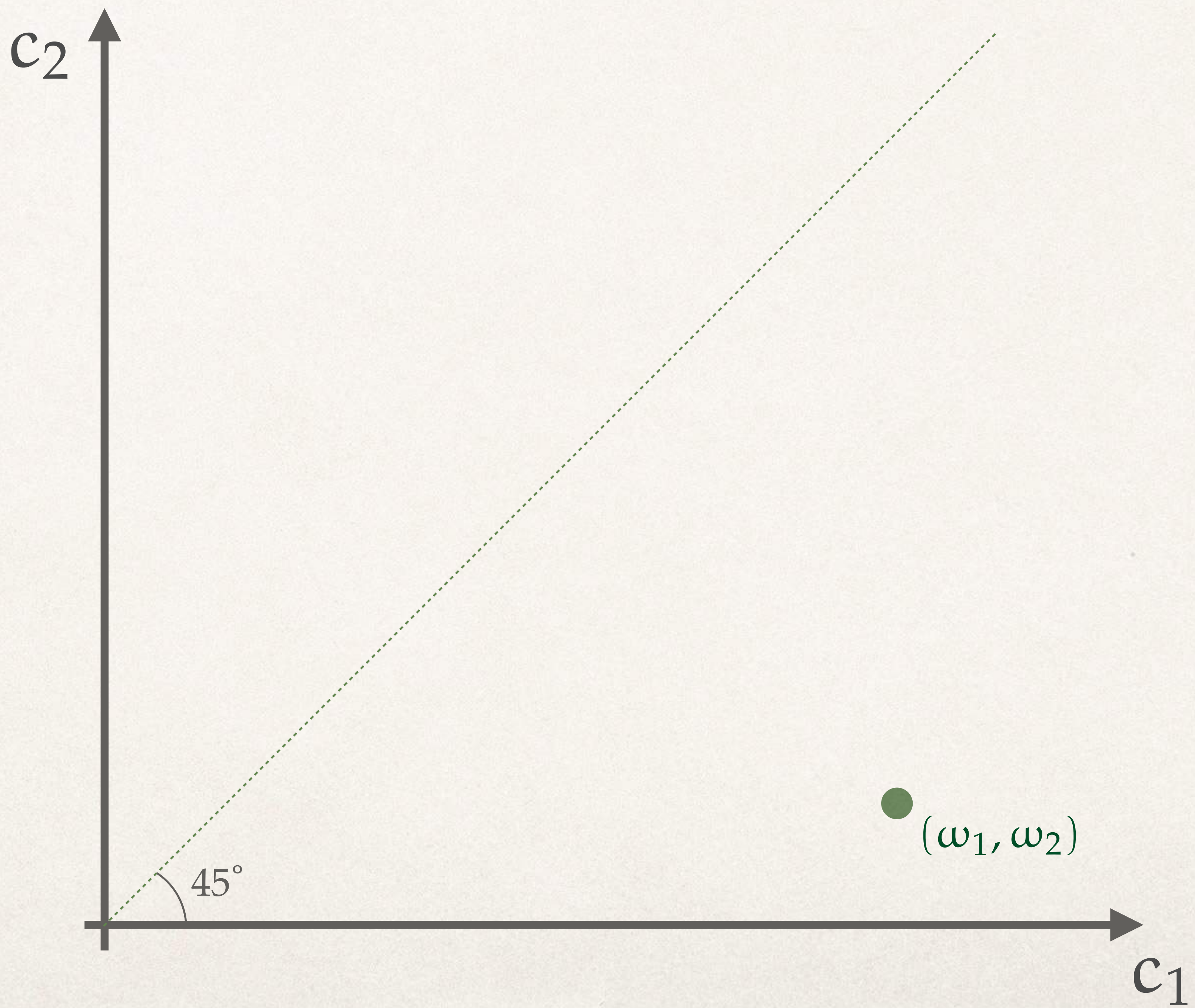

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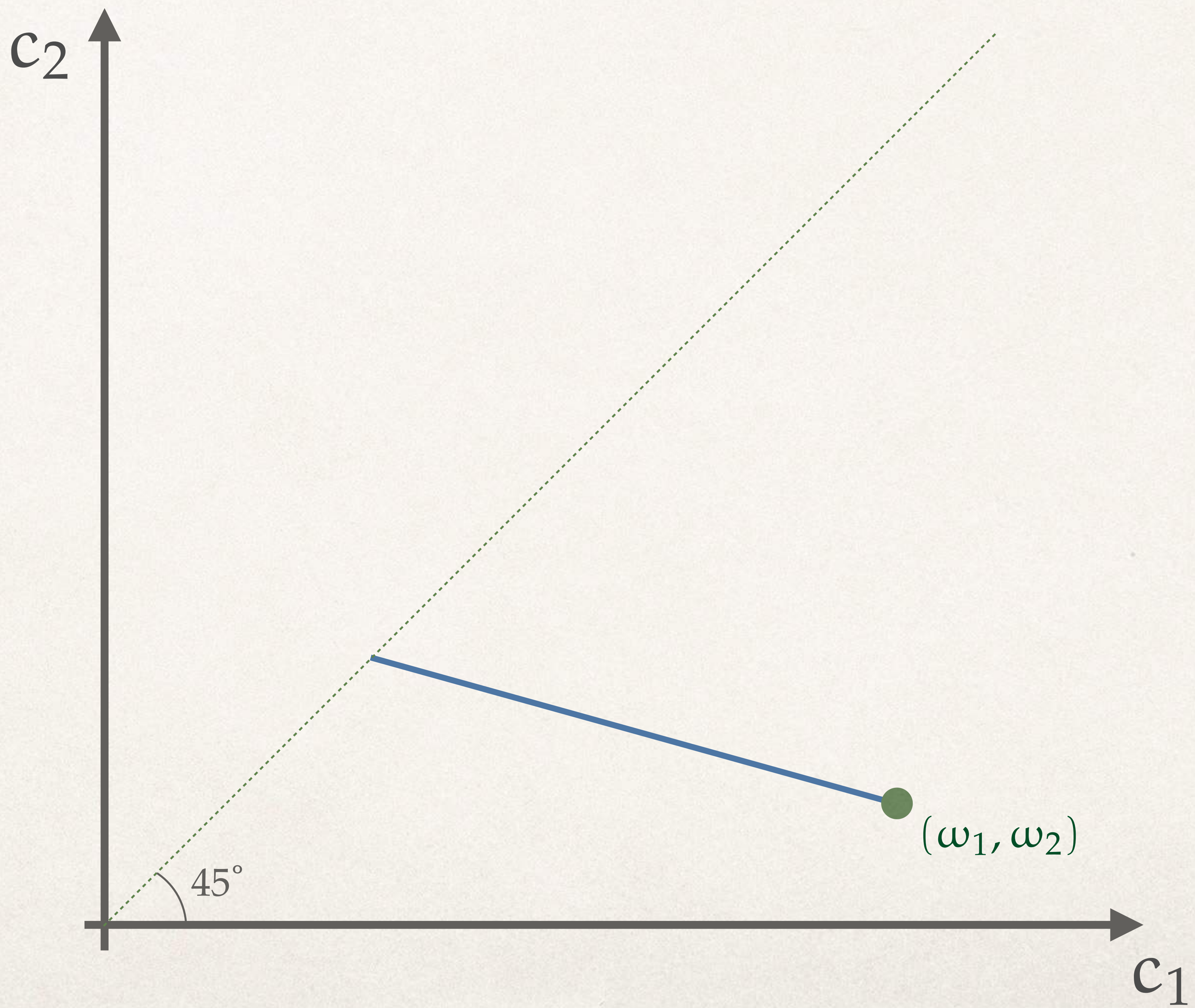

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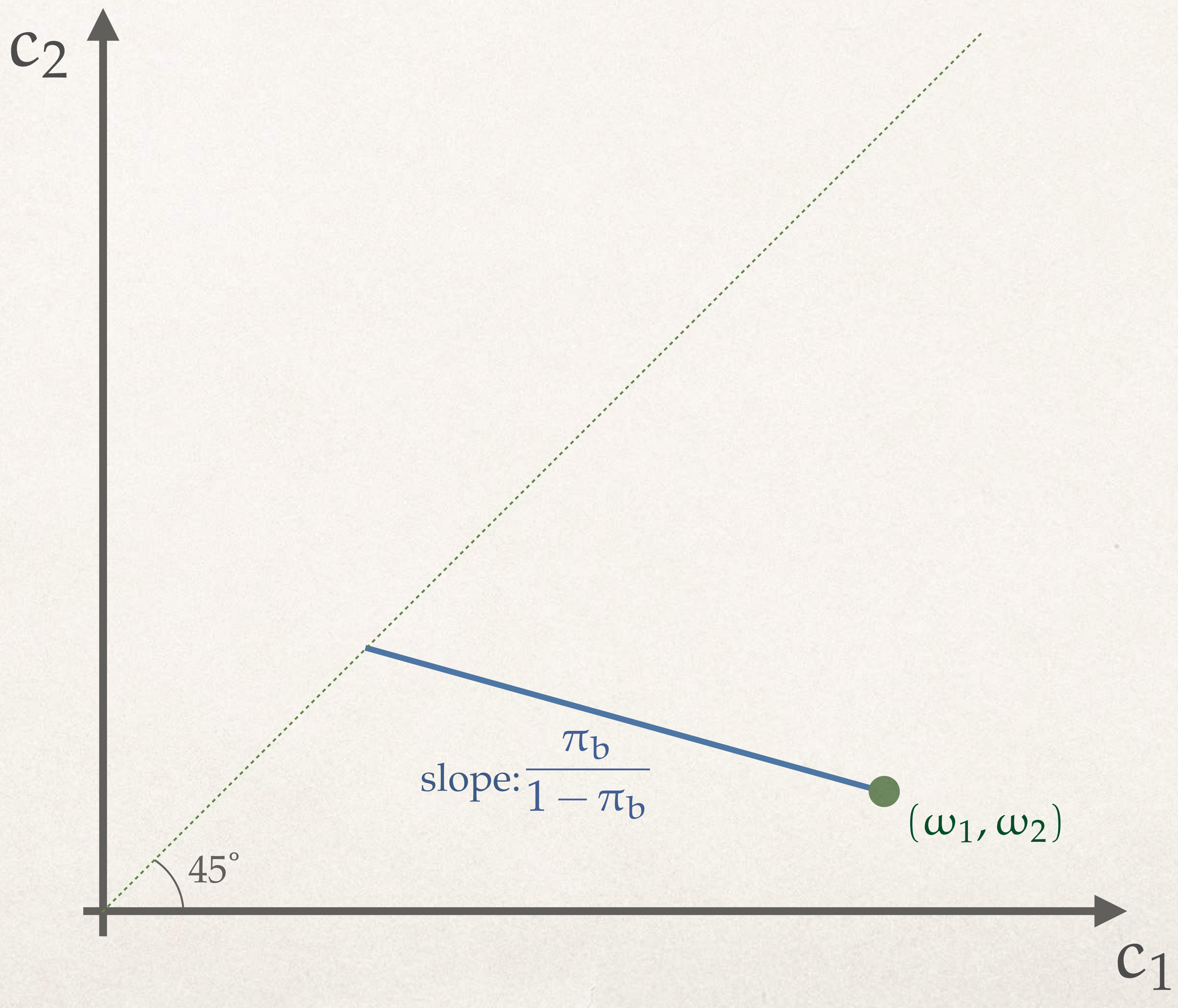

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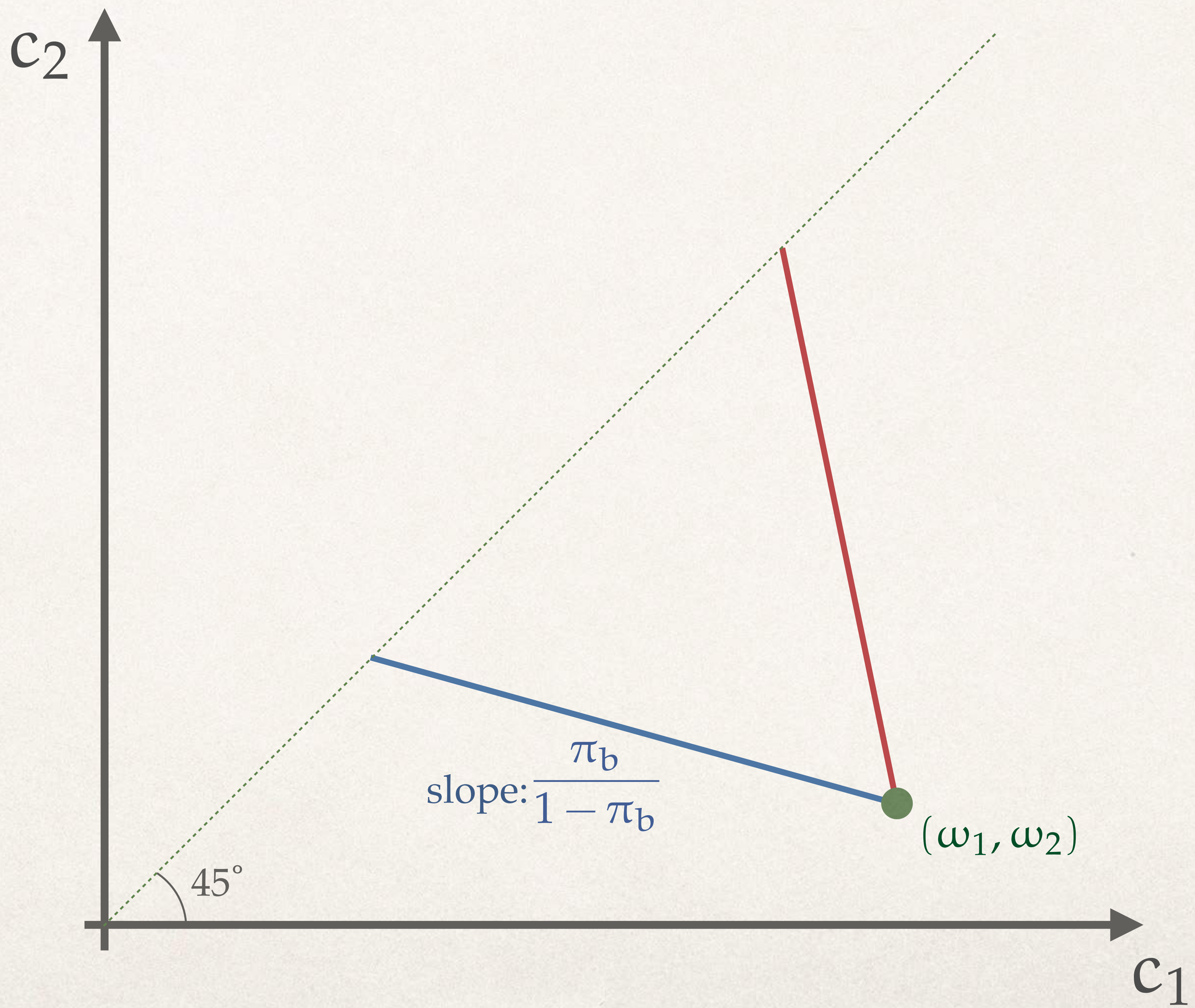

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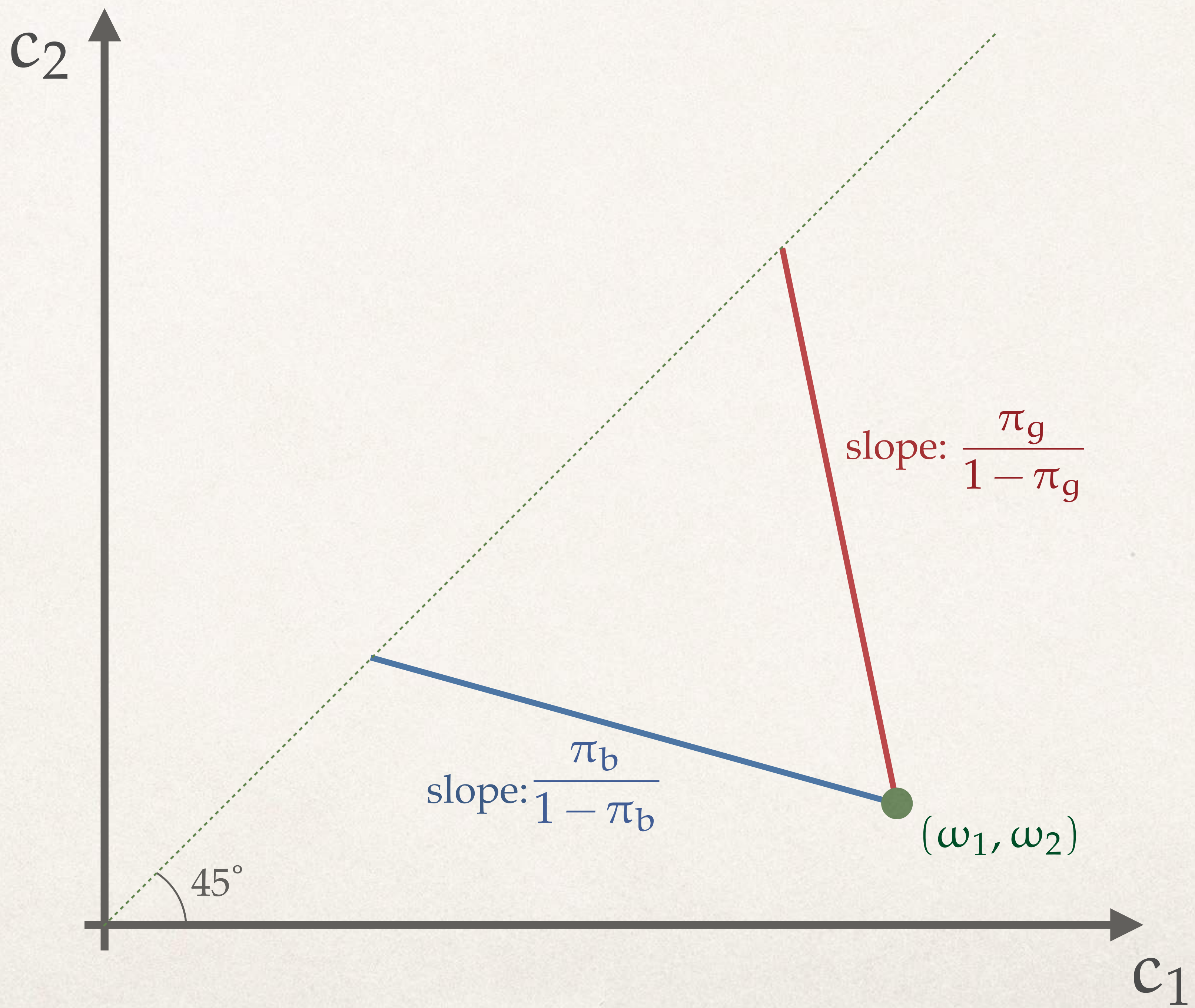

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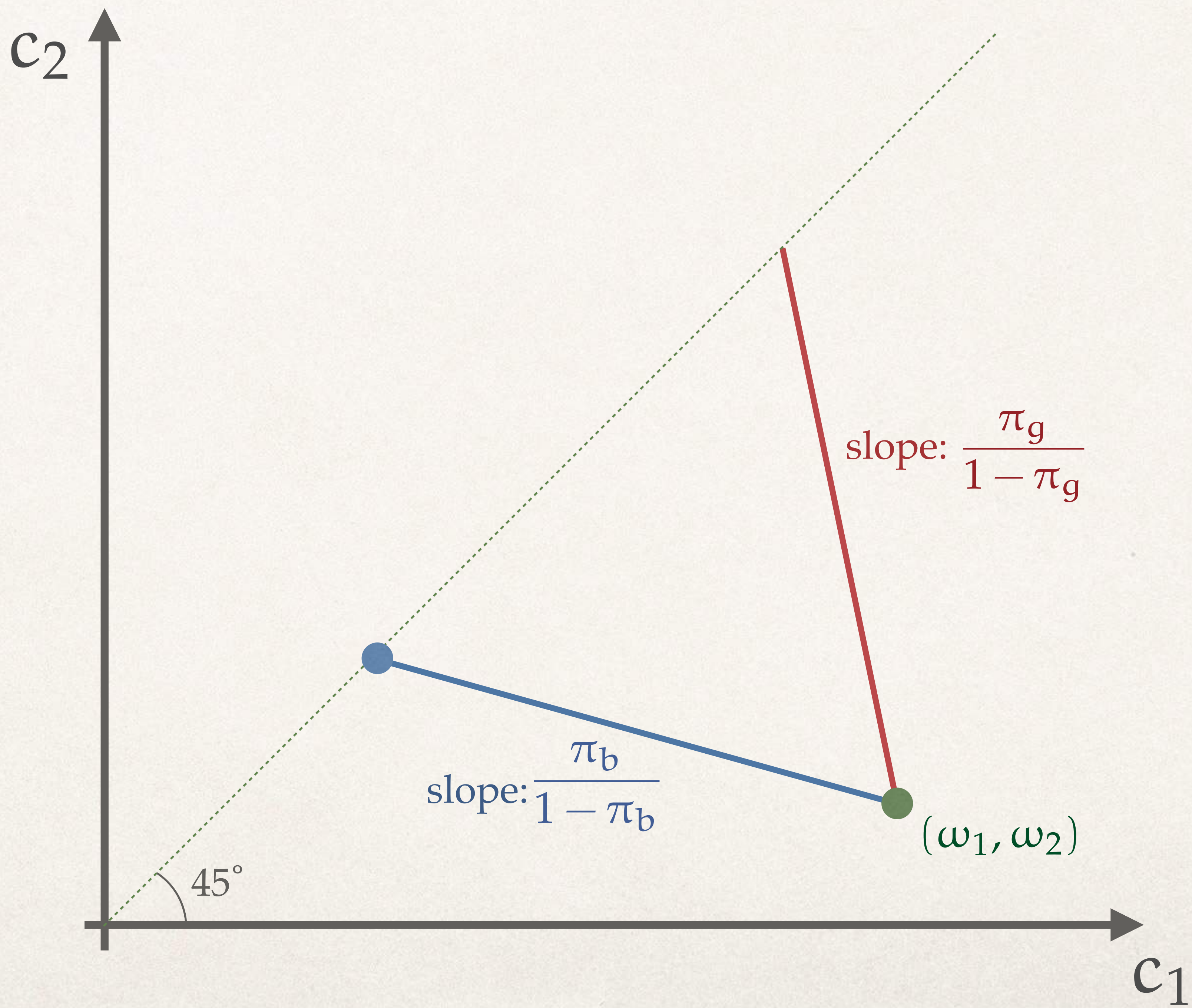

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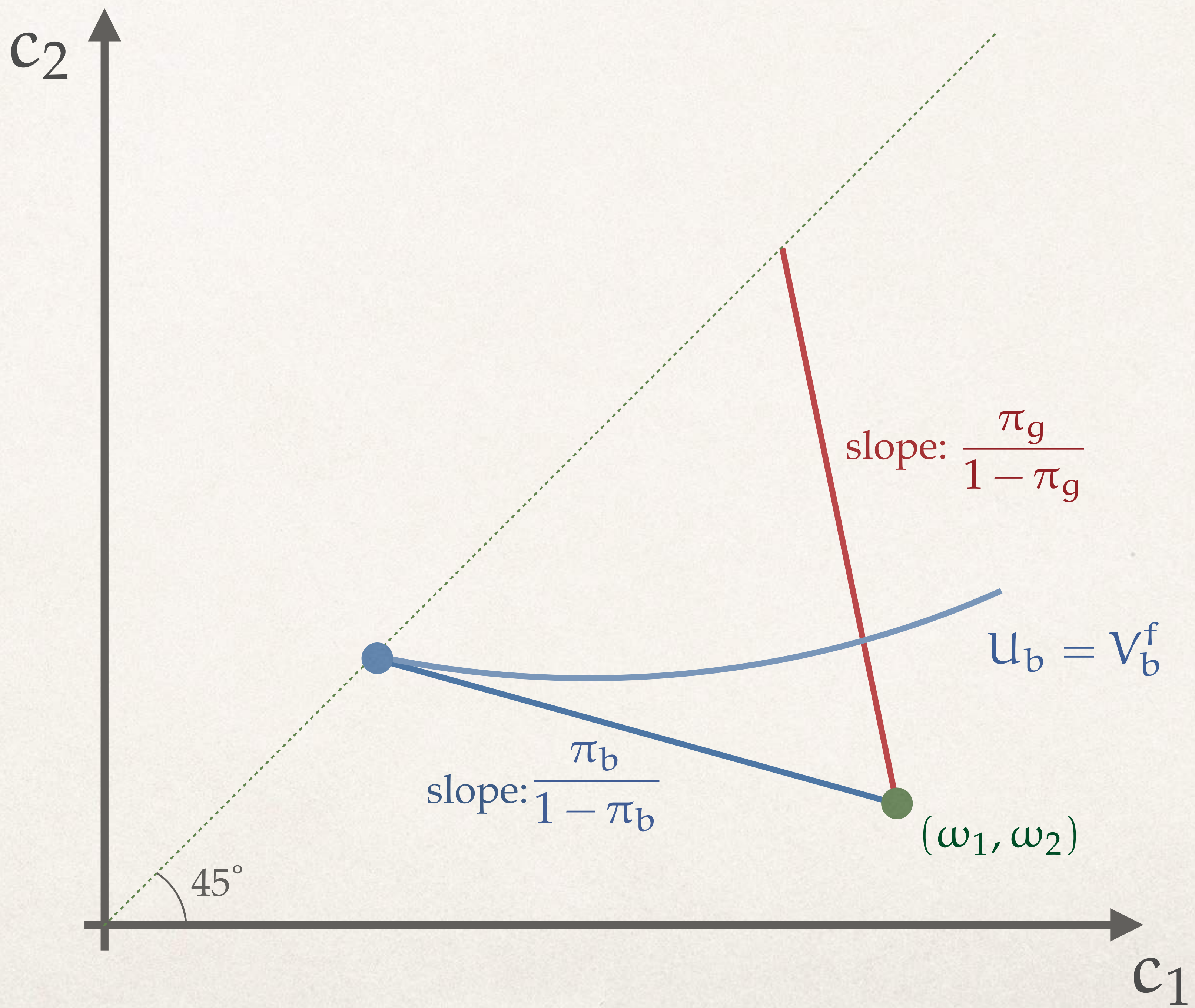

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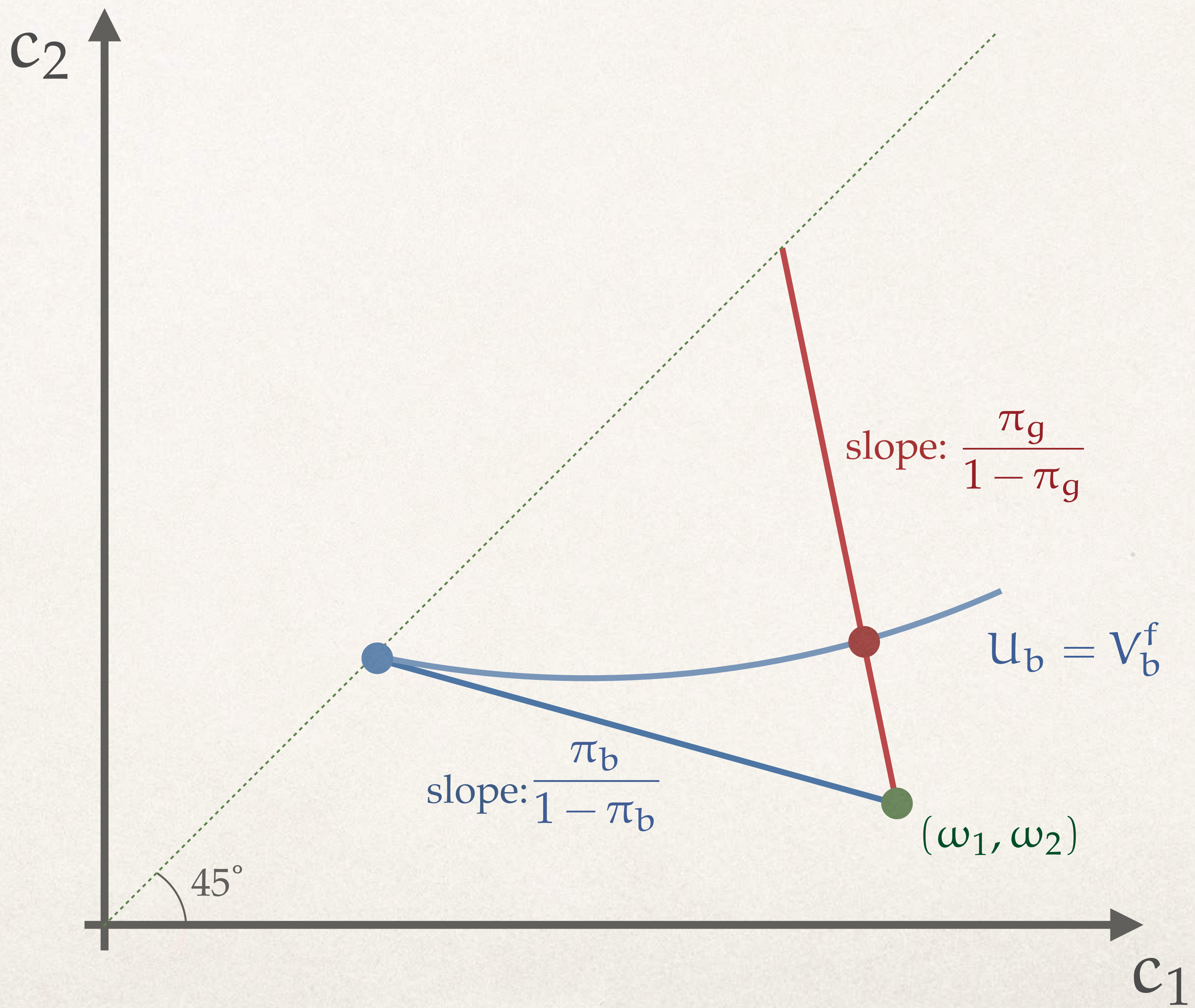

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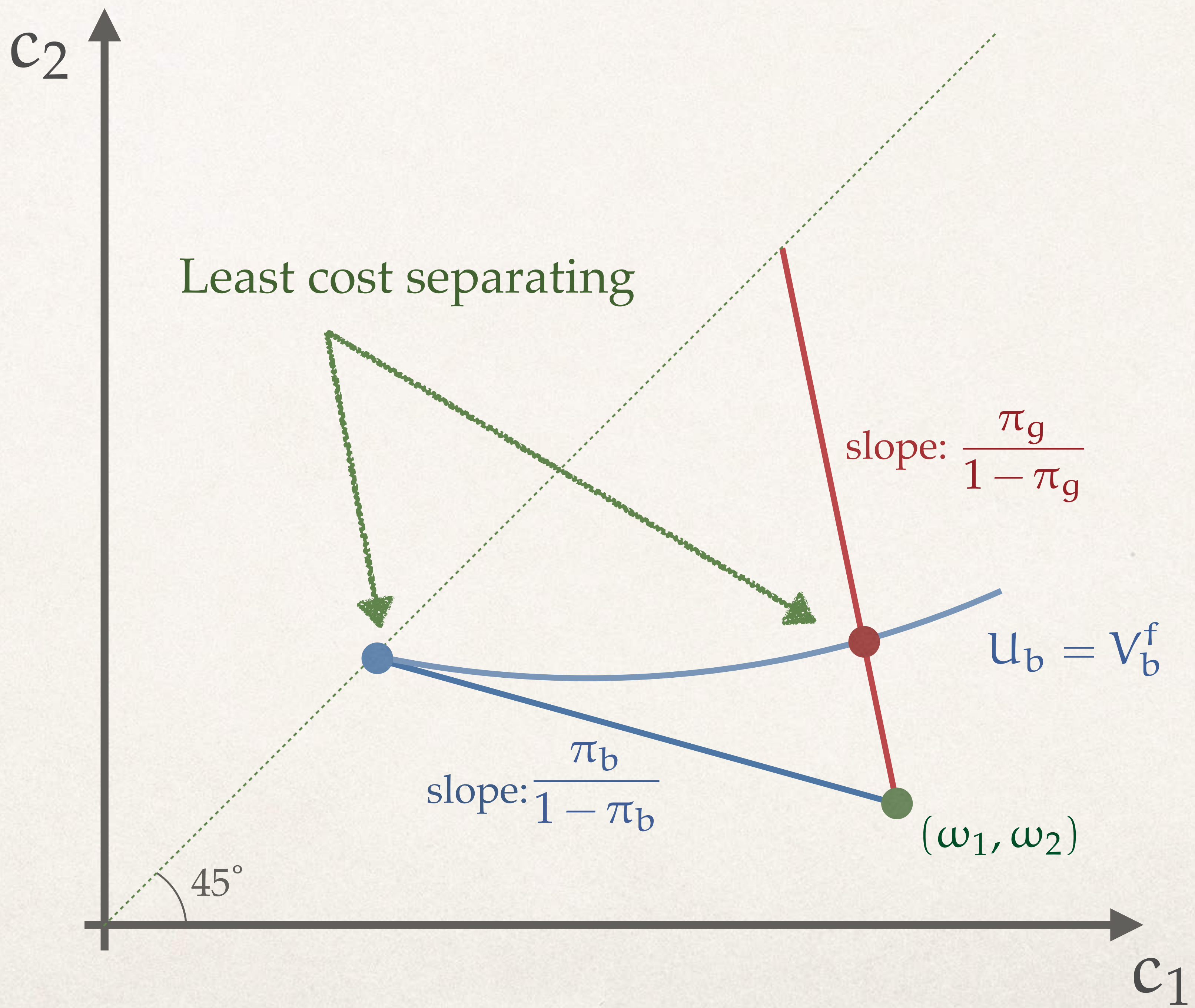
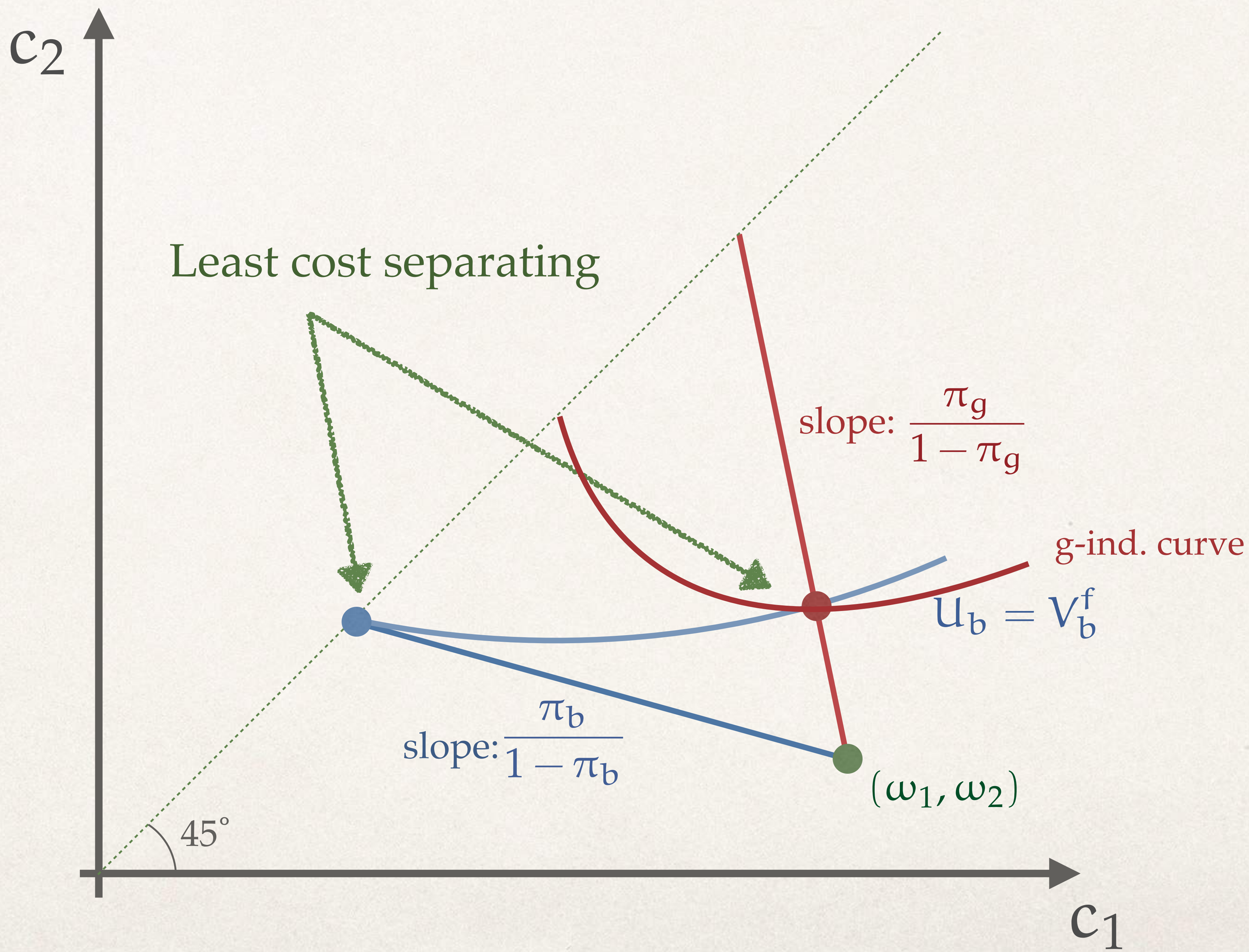


EFFICIENT ALLOCATIONS



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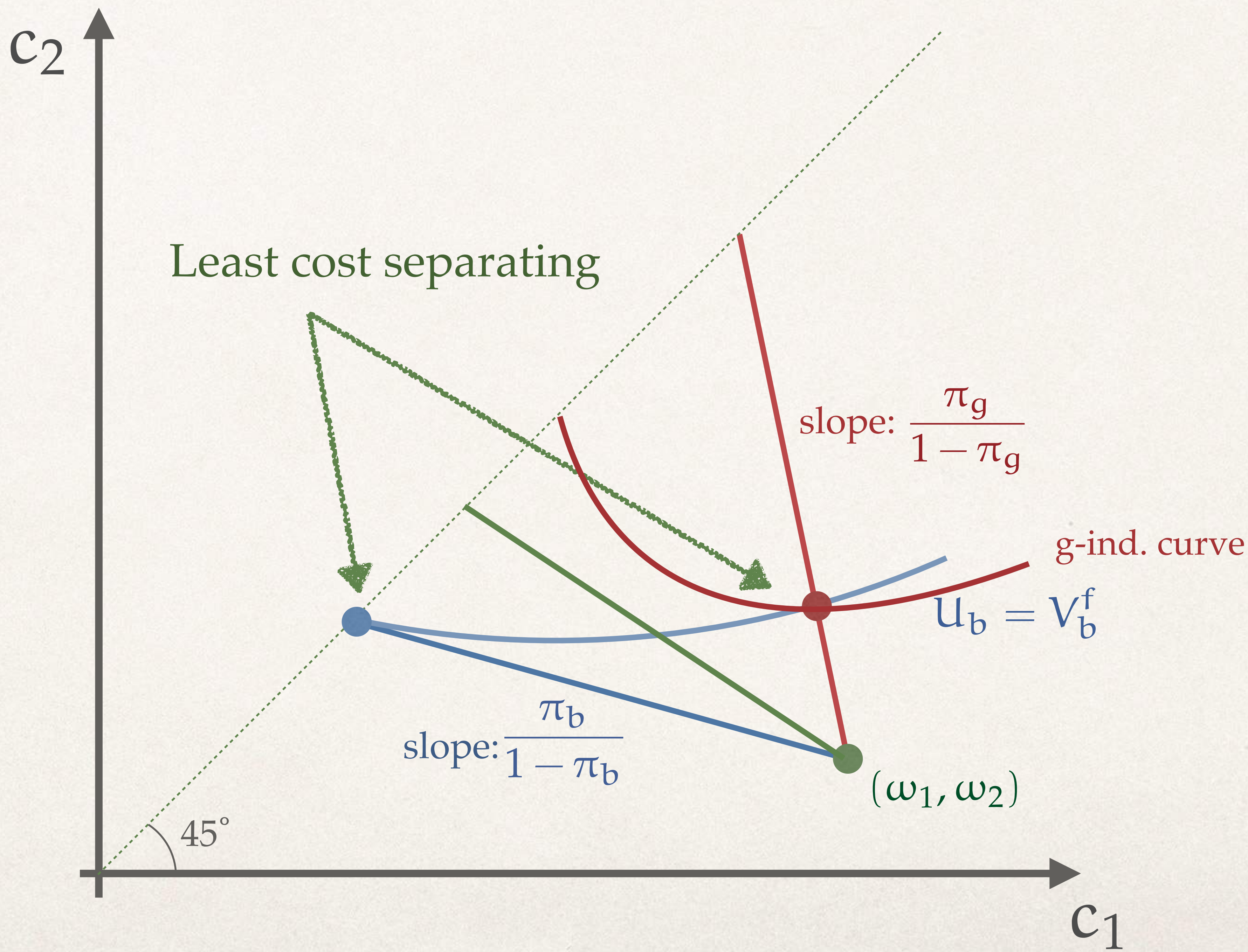

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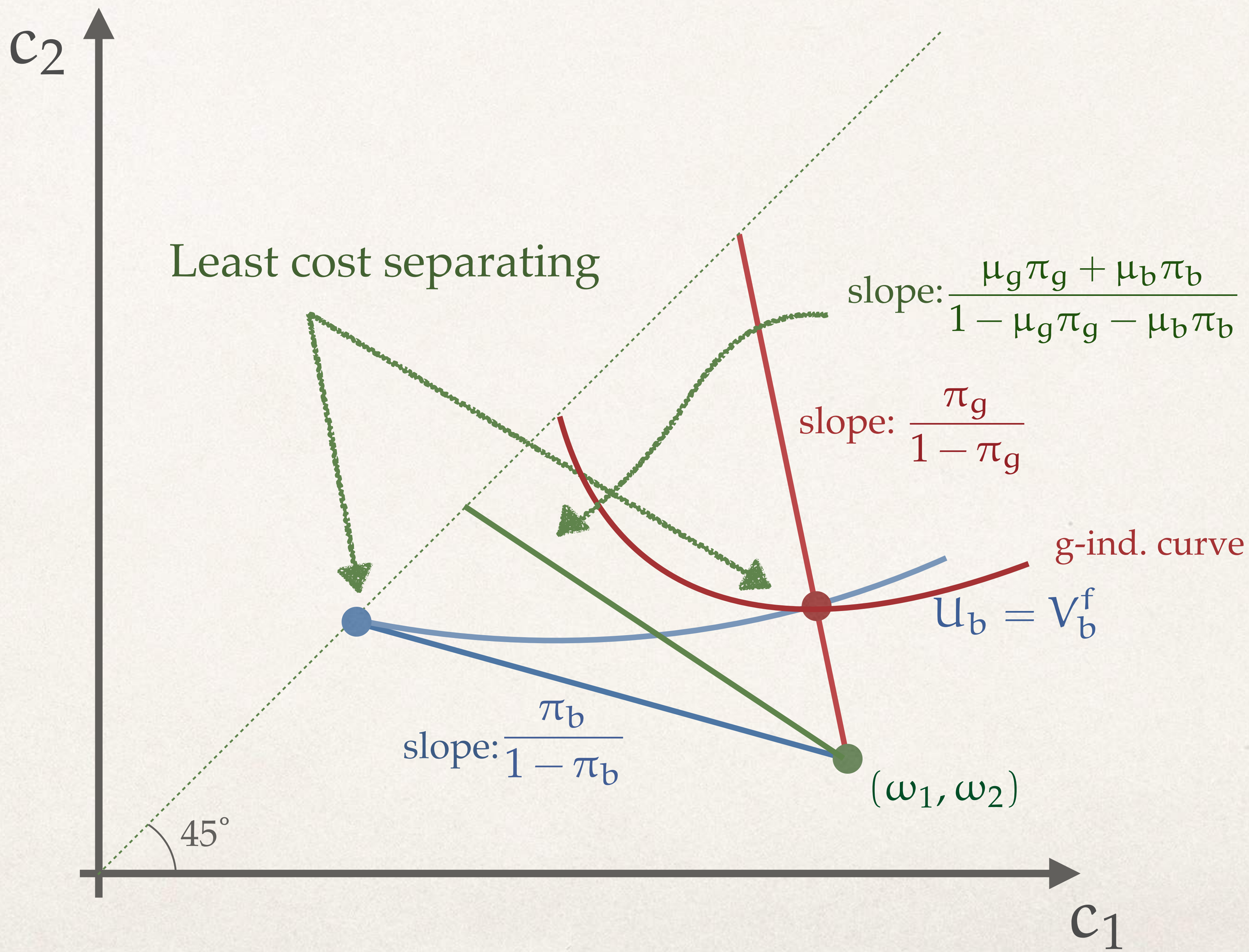

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- If $\frac{\lambda_g}{\lambda_g + \lambda_b} \geqslant \lambda^*$ then
  - ★ participation constraint is slack
  - ★ incentive constraint is binding
  - ★ cross-subsidization:
    - positive profits on g
    - negative profits on b

Efficient Allocations

- If $\frac{\lambda_g}{\lambda_g + \lambda_b} \geqslant \lambda^*$ then
  - participation constraint is slack
  - incentive constraint is binding
  - cross-subsidization:
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    - negative profits on b
- Focus only on $\mu_g \geqslant \lambda^*$

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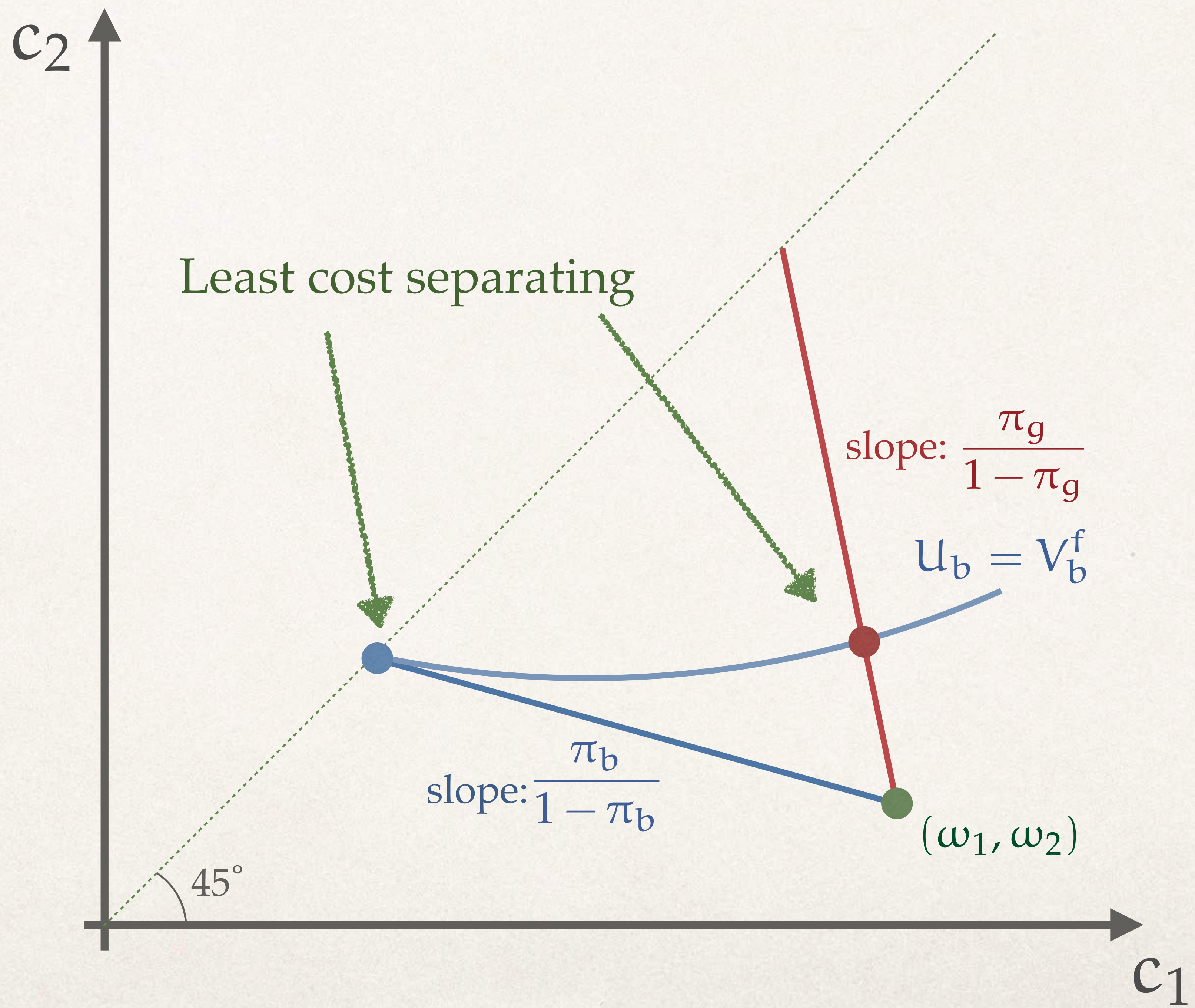

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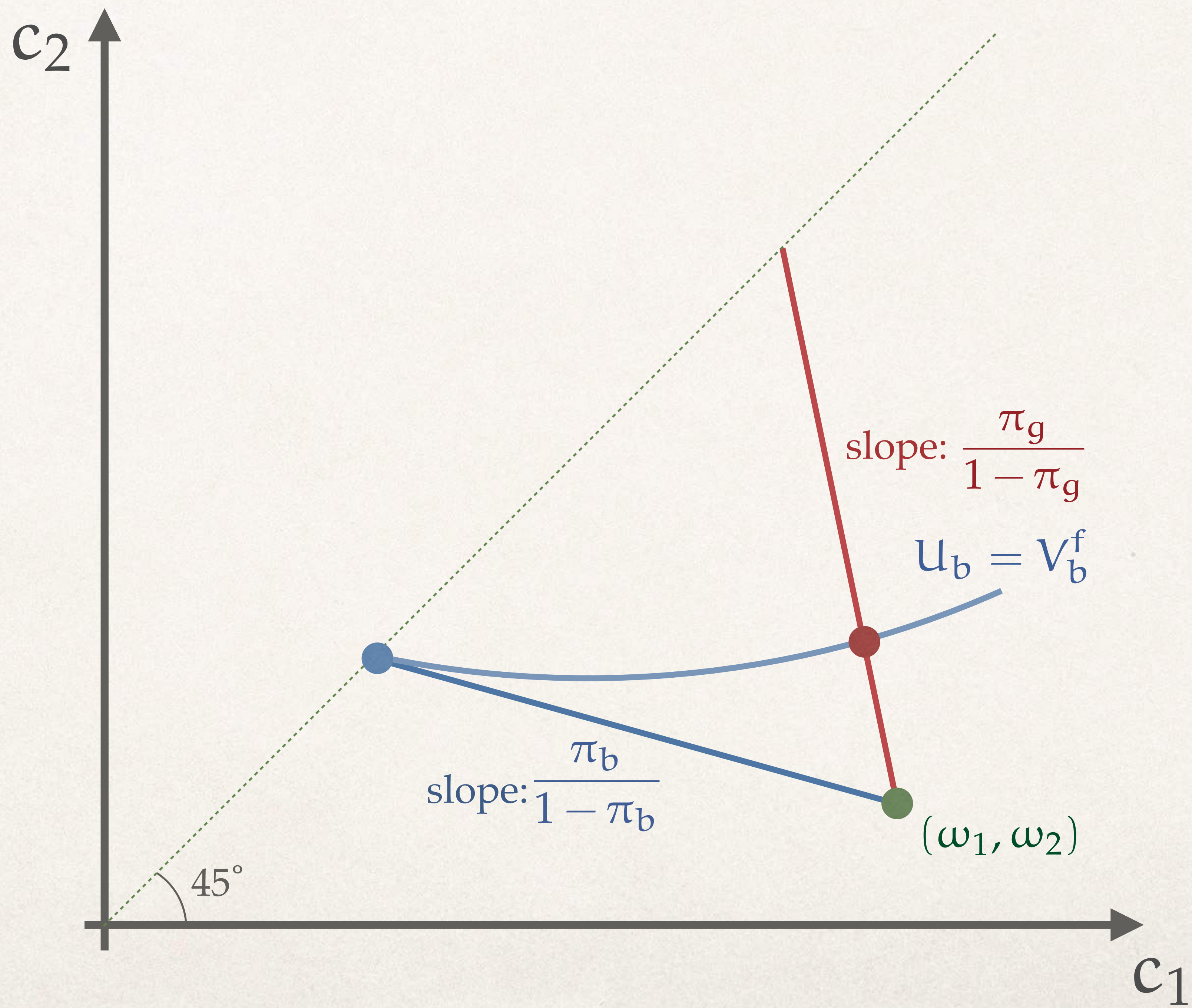

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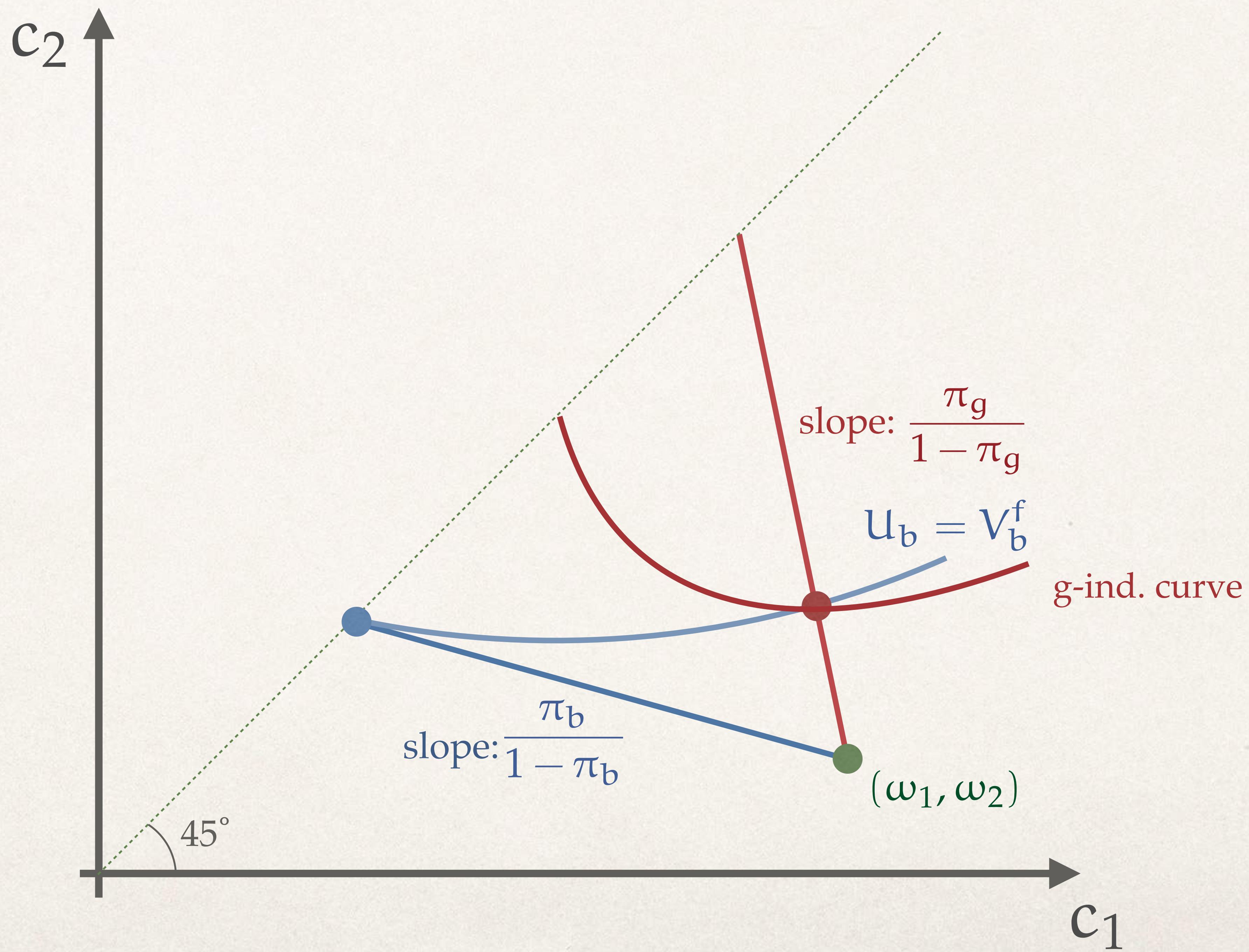

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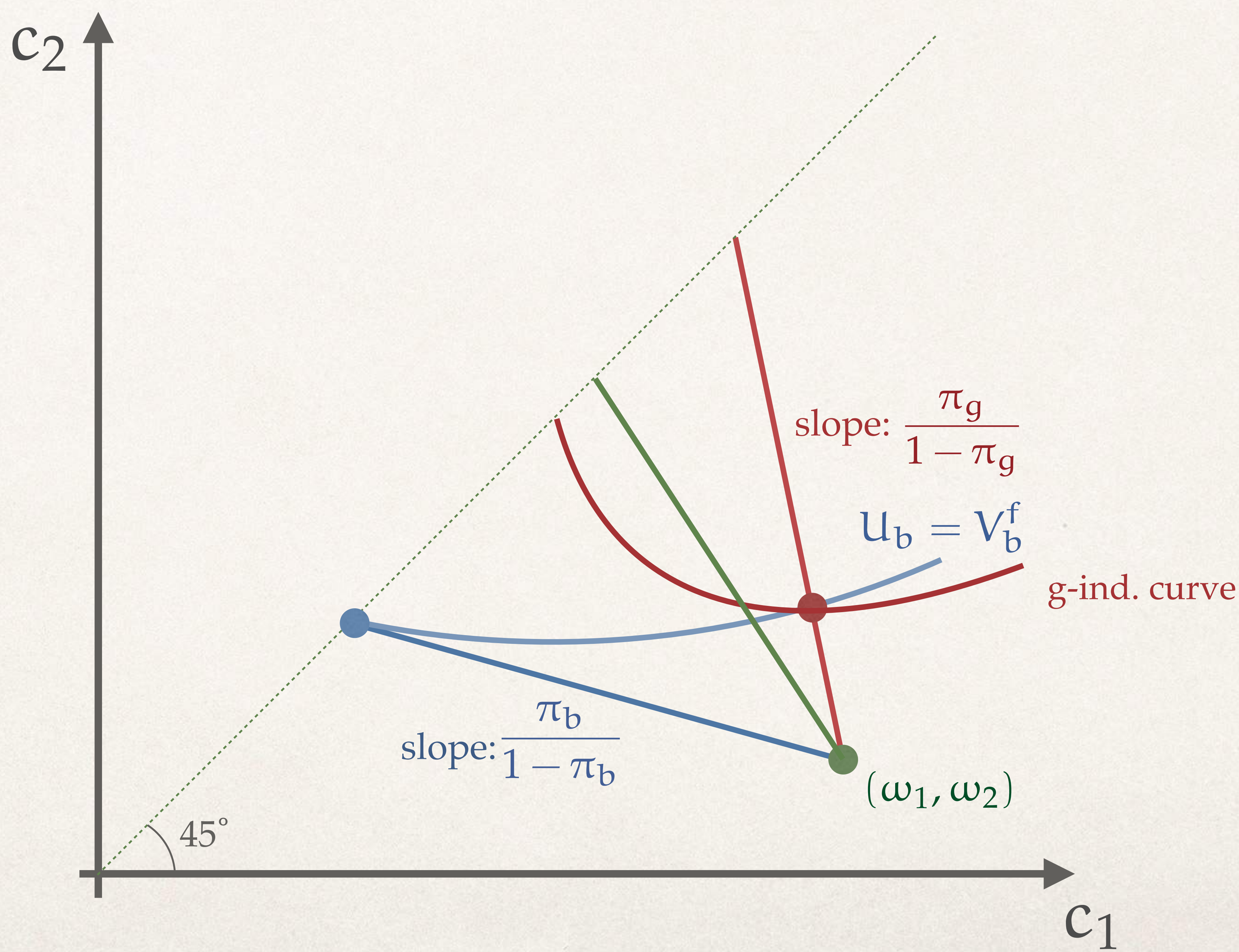

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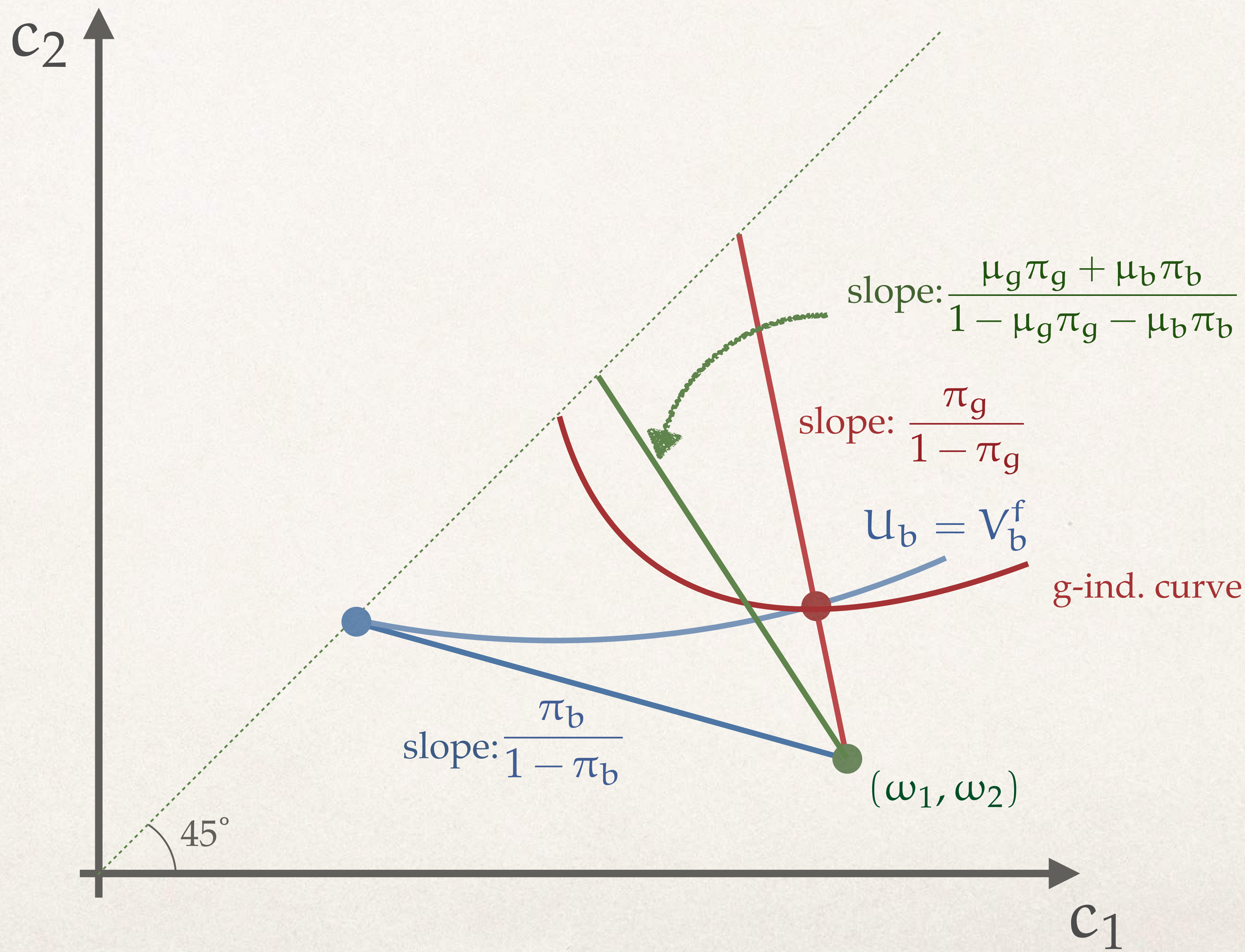

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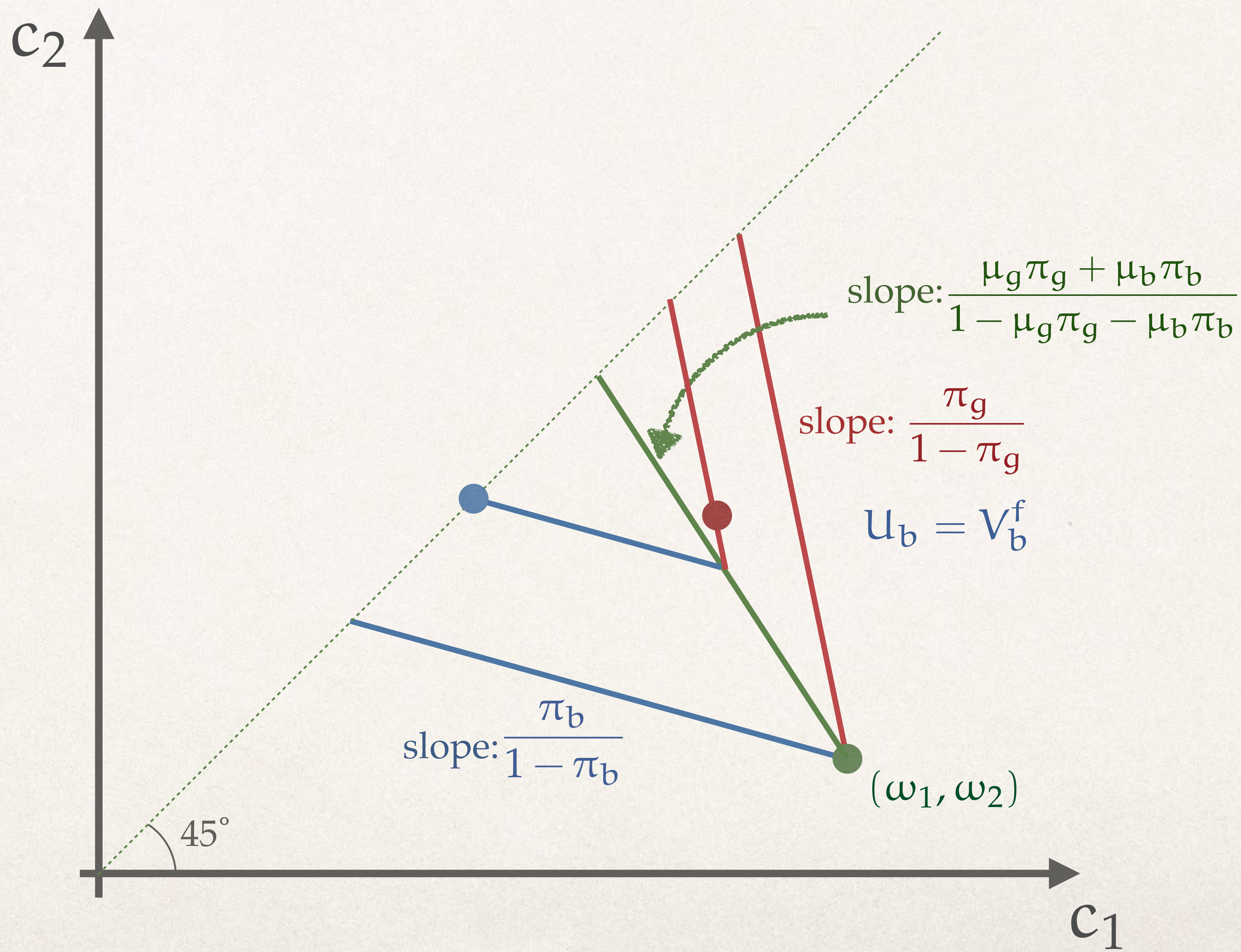
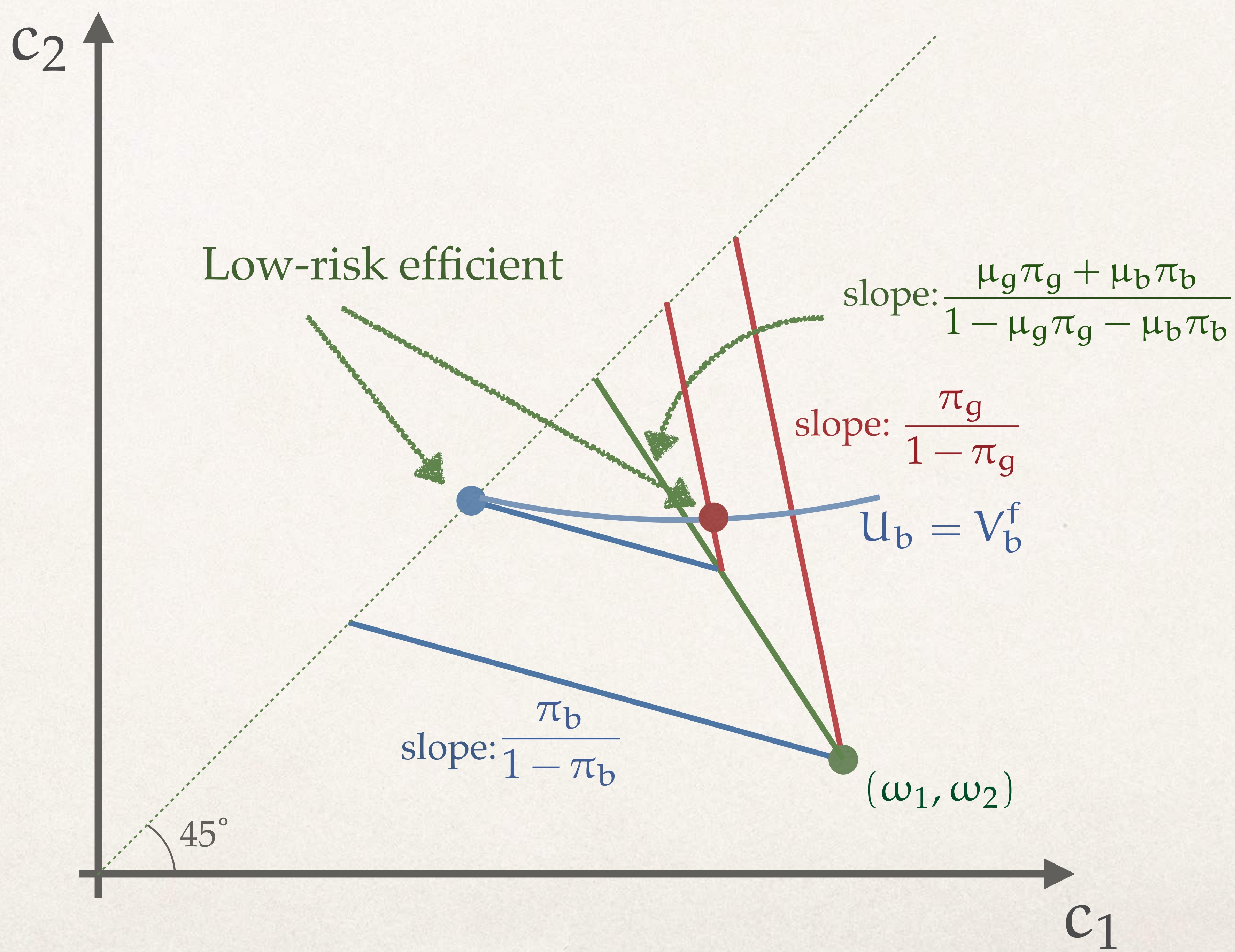


Efficient Allocations



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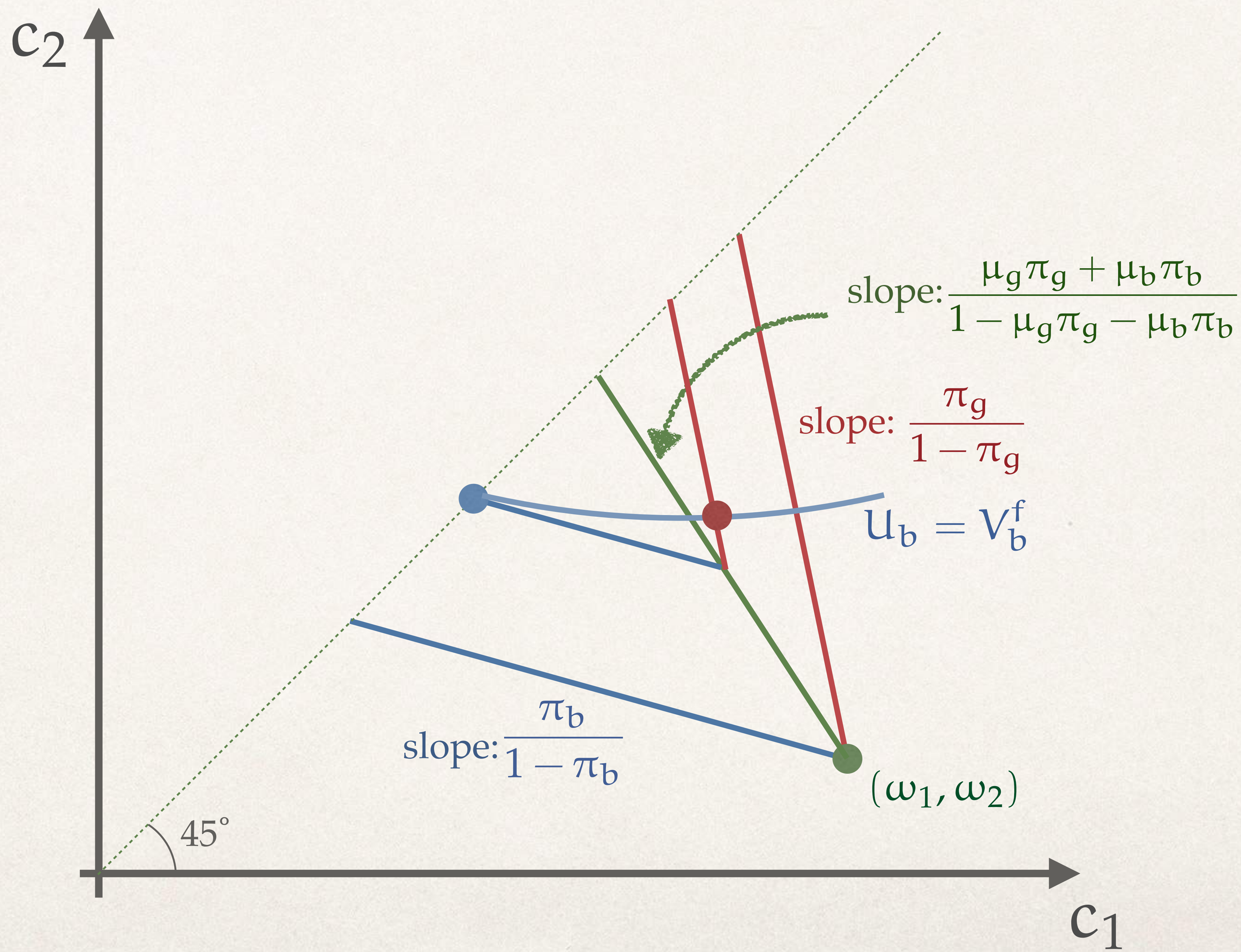

Efficient Allocations

- The functions $V_j^{eff}(\lambda_g, \lambda_b)$:
  - ⋆ increasing in $\frac{\lambda_g}{\lambda_g + \lambda_b}$ (constant below λ^*)
  - ⋆ necessarily discontinuous at $(0, 0)$
    - value at (0,0) is not defined
    - impossible to extend $V_j^{eff}(\lambda_g, \lambda_b)$ to (0,0) in a continuous way

Extensive Form Game

- Insurance companies move first:
  - ★ Offer menus

$$i \in \{1,2\}: \ \mathbf{c}^i(\lambda) = (c^i_{1l}(\lambda), c^i_{2l}(\lambda), c^i_{1h}(\lambda), c^i_{2h}(\lambda))$$

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- Households choose between the two firms
  - $\sigma^i_j(\mathbf{c}^1, \mathbf{c}^2)$: probability of choosing firm *i* by type *j*
- $\lambda^i = (\lambda^i_g, \lambda^i_b)$ measures of households

Rothschild-Stiglitz

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ROTHSCHILD-STIGLITZ

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  - ★ separating equilibrium
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  - ★ zero profits on each type
  - ★ least-cost-separating allocations

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  - ⋆ Dasgupta and Maskin (1986):

Rothschild-Stiglitz

- $\mu_g \leqslant \lambda^*$: Unique pure strategy equilibrium; interim efficient
- $\mu_g > \lambda^*$: no pure strategy equilibrium exists - same as in Riley
  - ⋆ Dasgupta and Maskin (1986):
    - mixed strategy equilibrium exists
    - inefficient (interim)

Our Equilibrium

Definition. A symmetric equilibrium is defined by a pair of menus $\mathbf{c}^i(\boldsymbol{\lambda}) : [0,1]^2 \mapsto \mathbb{R}^4, i = 1, 2$, together with households' strategies $\sigma^i_j : (\mathbf{c}^1, \mathbf{c}^2) \mapsto \Delta(\{1,2\}^2)$ such that:

i. households maximize: given any $\mathbf{c} = (\mathbf{c}^1, \mathbf{c}^2)$

$$\sigma^i_j(\mathbf{c}) \left[U_j(\sigma^i_g(\mathbf{c}), \sigma^i_b(\mathbf{c})) - U_j(\sigma^{-i}_g(\mathbf{c}), \sigma^{-i}_b(\mathbf{c})) \right] \geqslant 0$$

ii. firms maximize

$$\mathbf{c}^i \in \arg\max_{\mathbf{c}^i} \Pi(\mathbf{c}(\boldsymbol{\sigma}(\mathbf{c}^i, \mathbf{c}^{-i})))$$

Our Equilibrium

- Households take the choice of other households as given and optimize
- Firms take the decision of households into account; take the decision of other firm as given
- Restrict $\mathbf{c}^i(\boldsymbol{\lambda})$ to be continuous (except at (0,0)) and h.o.d. 0

Main Theorem

Theorem. The game has a unique symmetric equilibrium that coincides with the low-risk efficient allocation.

Proof in Steps

- Construct equilibrium strategies
- Show result for a restricted set of equilibrium strategies
- General result

Mirror strategies

- Construct equilibrium strategies from low-risk efficient allocation

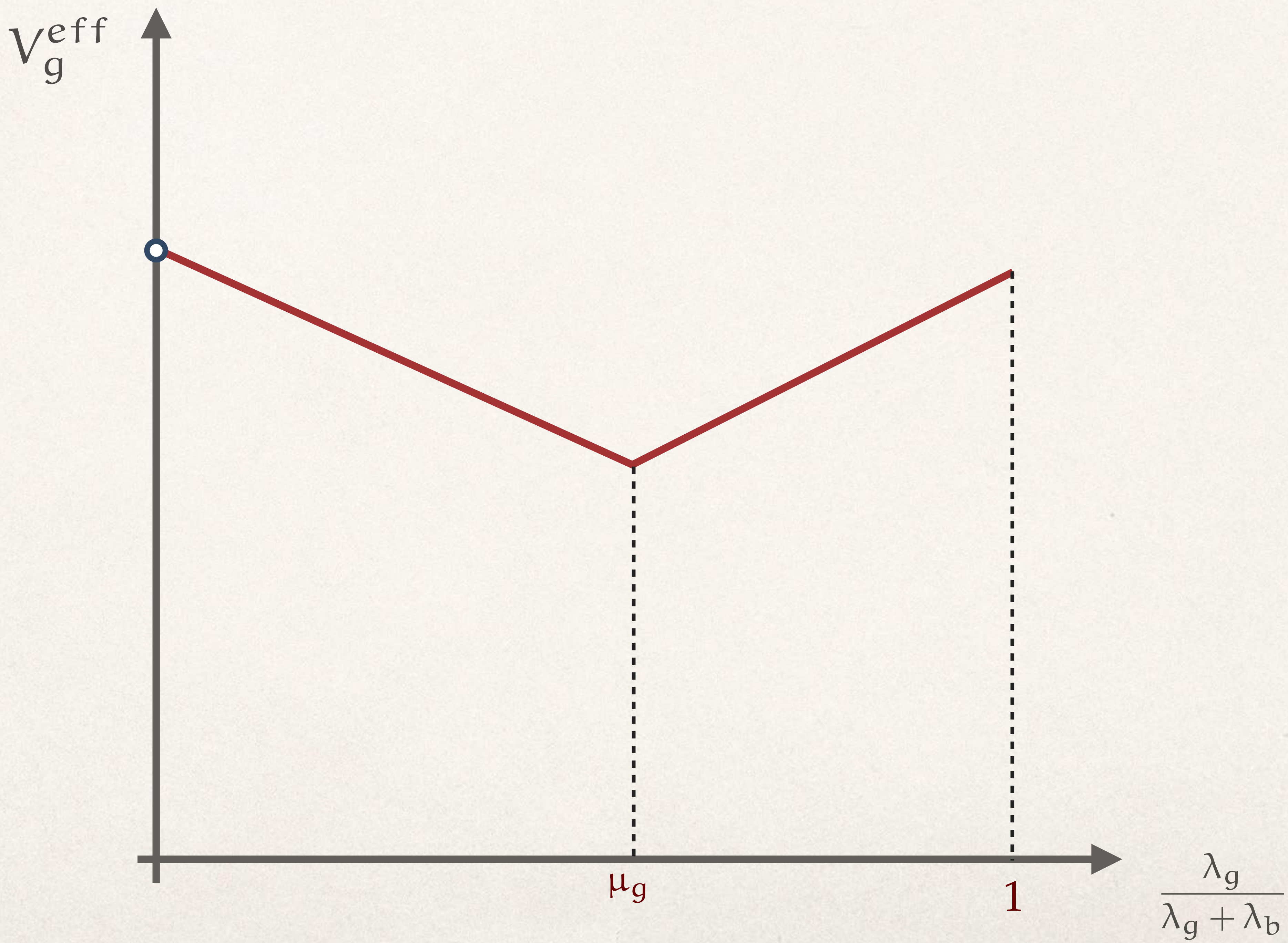
$$V_j^*(\boldsymbol{\lambda}) = \max\left\{V_j^{eff}(\boldsymbol{\lambda}), V_j^{eff}(\boldsymbol{\lambda}^c)\right\}, j = l, h$$

where

$$\boldsymbol{\lambda}^c = (\mu_h - \lambda_h, \mu_l - \lambda_l)$$

- Associated menus are given by $\mathbf{c}^*(\boldsymbol{\lambda})$

Mirror Strategies



Proof - First Step

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- First step: restrict strategies

$$S = \{\mathbf{c}(\boldsymbol{\lambda})\,; \text{The subgame with } (\mathbf{c}(\boldsymbol{\lambda}), \mathbf{c}^*(\boldsymbol{\lambda})) \text{ has an equilibrium}\}$$

PROOF - FIRST STEP

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Proposition 1. Consider the restricted game in which each firm offers menus $\mathbf{c} \in S$. Then the low-risk efficient allocation is an equilibrium outcome of the game.

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Proposition 1. Consider the restricted game in which each firm offers menus $\mathbf{c} \in S$. Then the low-risk efficient allocation is an equilibrium outcome of the game.

- Why restriction: every subgame is a discontinuous non-atomic game:
  - ⋆ equilibrium does not necessarily exist!

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Proof of Proposition 1

- Idea of proof:
  - Suppose that firm 2 - incumbent - offers the mirror strategy menu $\mathbf{c}^*(\boldsymbol{\lambda})$
  - Firm 1 - deviant offers $\hat{\mathbf{c}}(\boldsymbol{\lambda}) \in S$
  - Equilibrium of the subgame represented by $\lambda^1 \neq (0,0)$ at firm 1

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mirror strategies

- firm 1 cannot make positive profits

Market Design for Insurance

- Mutualization is a form of market design
- we assumed all costs of mutualization away: cost of capital goes up
- Market design question: how do we design health exchanges
  - Obamacare; similar setups in Netherlands and Switzerland
  - Recent example: Handel, Hendel and Whinston 2015

Empirics of Adverse Selection

Empirics of Adverse Selection

- Key question: Is there adverse selection in a certain market?
- Maybe from trade volume? Trade volume fluctuates for all sorts of reasons
- Perhaps from cross-sectional implication of the models?

Tests of Adverse Selection in Insurance Markets

- In all of the models discussed, quantity and risk-types are positively correlated: higher risk individuals purchase more insurance
- Important caveat: what matters for pricing is the information set of insurers; always a concern
- Salanie and Chiappori, 2002: look at young drivers in France (less than 2 years of driving experience)

Test of Adverse Selection

- Key regressions - two probits (they also perform fancier non-parametric tests):

$$\begin{aligned} y_i &= \mathbf{1}(X_i\beta + \epsilon_i) \\ z_i &= \mathbf{1}(X_i\gamma + \eta_i) \end{aligned}$$

  y_i: choice of coverage; z_i: occurrence of accident; X_i: individual characteristic

- Theory: with adverse selection ϵ_i and η_i should be negatively correlated; without it they should not be!

- Result: they are not correlated

Test of Adverse Selection

- Finkelestein and McGarry 2006: This result does not mean no adverse selection; need direct tests

- They looked at long-term care insurance: insurance against the risk of going to a nursing home
- They had access to data from a survey:
  "Of course nobody wants to go to a nursing home, but sometimes it becomes necessary. What do you think are the chances that you will move to a nursing home in the next five years?"
- They show that the answer to this is correlated with the outcome beyond variables observed by the insurance companies

Tests of Adverse Selection

- Run probits again:

$$Prob(\text{CARE} = 1) = \Phi(X\beta_1 + \beta_2 \boldsymbol{B}).$$

$$Prob(\text{LTCINS} = 1) = \Phi(X\delta_1 + \delta_2 \boldsymbol{B}).$$

- **B**: beliefs about the needs

Tests of Adverse Selection

TABLE 1—RELATIONSHIP BETWEEN INDIVIDUAL BELIEFS AND SUBSEQUENT NURSING HOME USE

	No controls	Control for insurance company prediction		Control for application information
	(1)	(2)	(3)	(4)
Individual prediction	0.091***		0.043**	0.037*
	(0.021)		(0.020)	(0.019)
Insurance company prediction		0.400***	0.395***	
		(0.020)	(0.021)	
pseudo-R^2	0.005	0.097	0.099	0.183
N	5,072	5,072	5,072	4,780

TABLE 2—RELATIONSHIP BETWEEN INDIVIDUAL BELIEFS AND INSURANCE COVERAGE

	No controls	Control for insurance company prediction		Control for application information
	(1)	(2)	(3)	(4)
Individual prediction	0.086***		0.099***	0.083***
	(0.017)		(0.017)	(0.016)
Insurance company prediction		−0.125***	−0.140***	
		(0.023)	(0.023)	
pseudo-R^2	0.007	0.010	0.019	0.079
N	5,072	5,072	5,072	4,780

Tests of Adverse Selection

- Replicating Chiappori-Salanie's Test:

TABLE 3—THE RELATIONSHIP BETWEEN LONG-TERM CARE INSURANCE AND NURSING HOME ENTRY

	No controls (1)	Controls for insurance company prediction (2)	Controls for application information (3)
Correlation coefficient from bivariate probit of LTCINS and CARE	−0.105***	−0.047	−0.028
	(p = 0.006)	(p = 0.25)	(p = 0.51)
Coefficient from probit of CARE on LTCINS	−0.046***	−0.021	−0.014
	(0.015)	(0.016)	(0.016)
N	5,072	5,072	4,780

Tests of Adverse Selection

- So how do we make sense of these?
- Multi-dimensional heterogeneity: perhaps low-risk individuals are also highly risk-averse. So they would like to purchase a lot of coverage for a lower probability event